VISUAL PHYSICS

SHORT NOTES

CHAPTER

Capacitors Dielectrics

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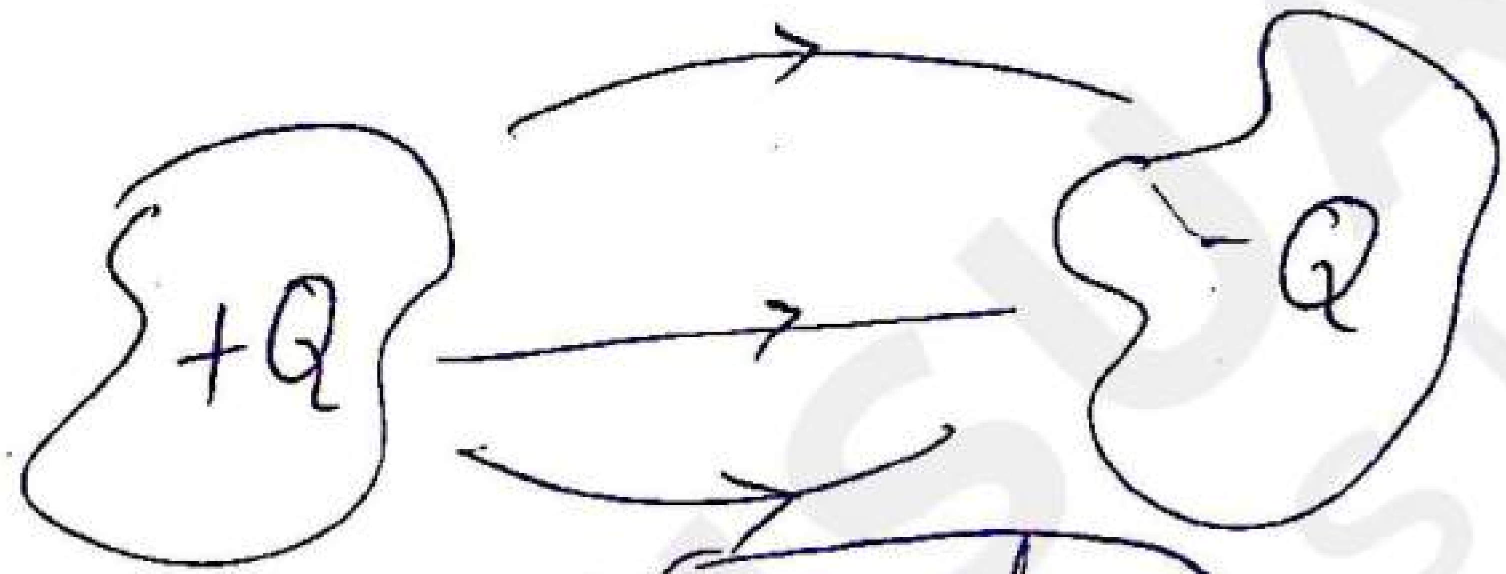
CAPACITOR AND CAPACITANCE

Capacitor → Any two conductors seperated by an insulator form a capacitor.

dielectric



Capacitance

$\boxed{Q = CV}$ → potential difference between the conductors

charge stored in capacitor

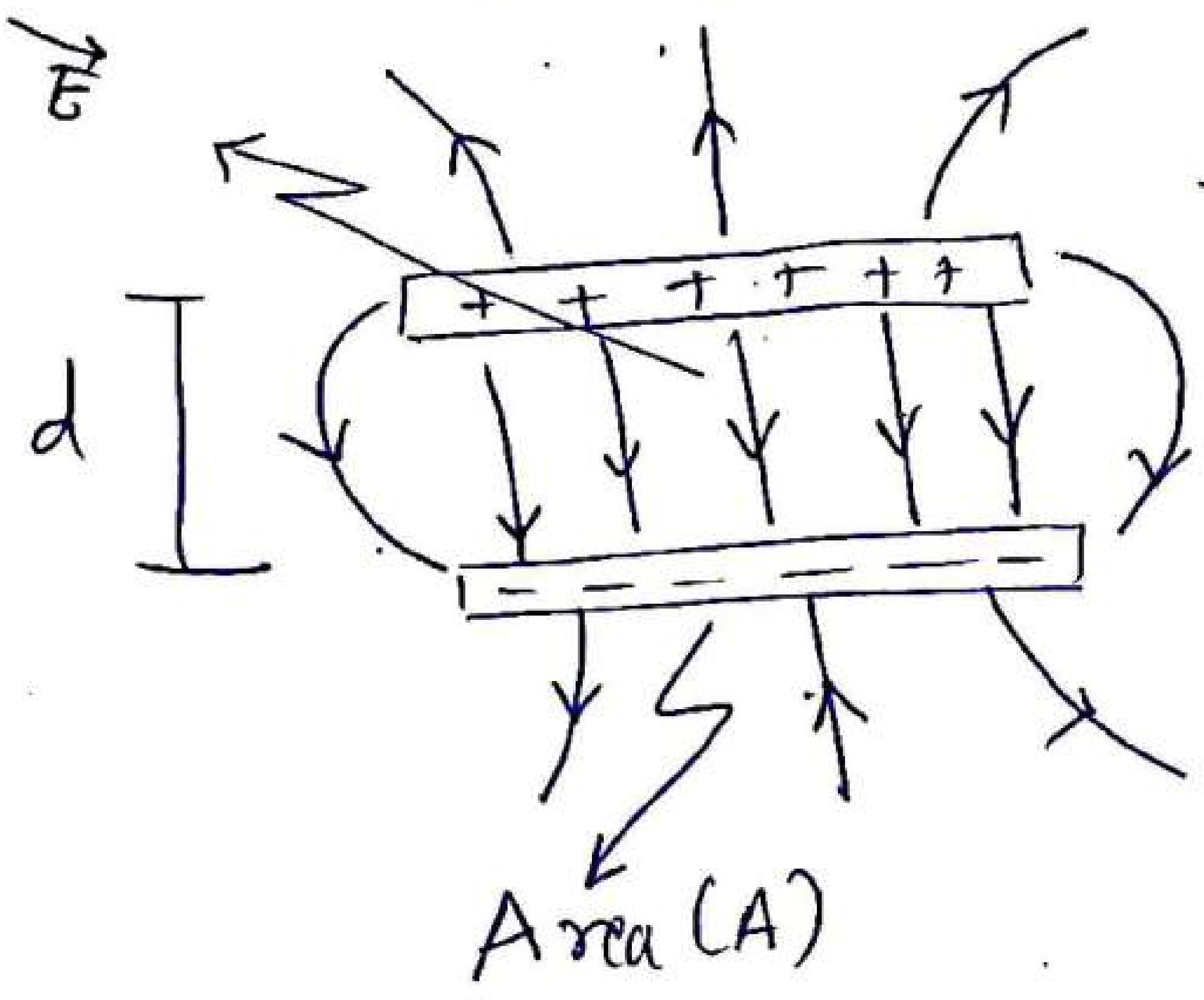
depends on dimension of capacitor

Capacitance → means the ability of capacitor to store charge for a given potential difference.

unit → Farad

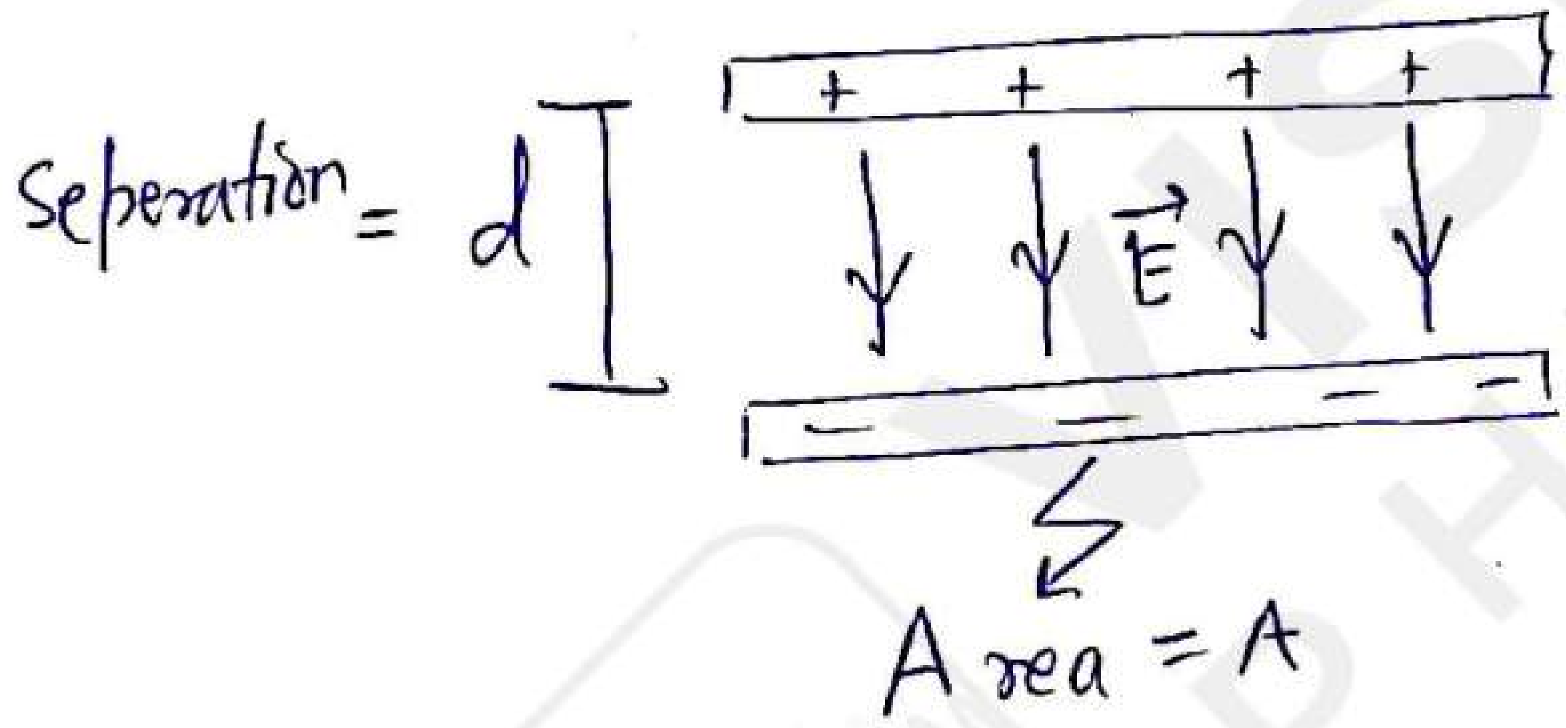
★ A single conductor also acts as a capacitor by assuming that the other conductor is placed at infinity.

Parallel plate capacitor:



→ fringe effect for the case when d (seperation) is less

When the seperation is sufficient: (no fringe effect)



$$\vec{E} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\sigma}{\epsilon_0}}$$

V = potential difference = Ed

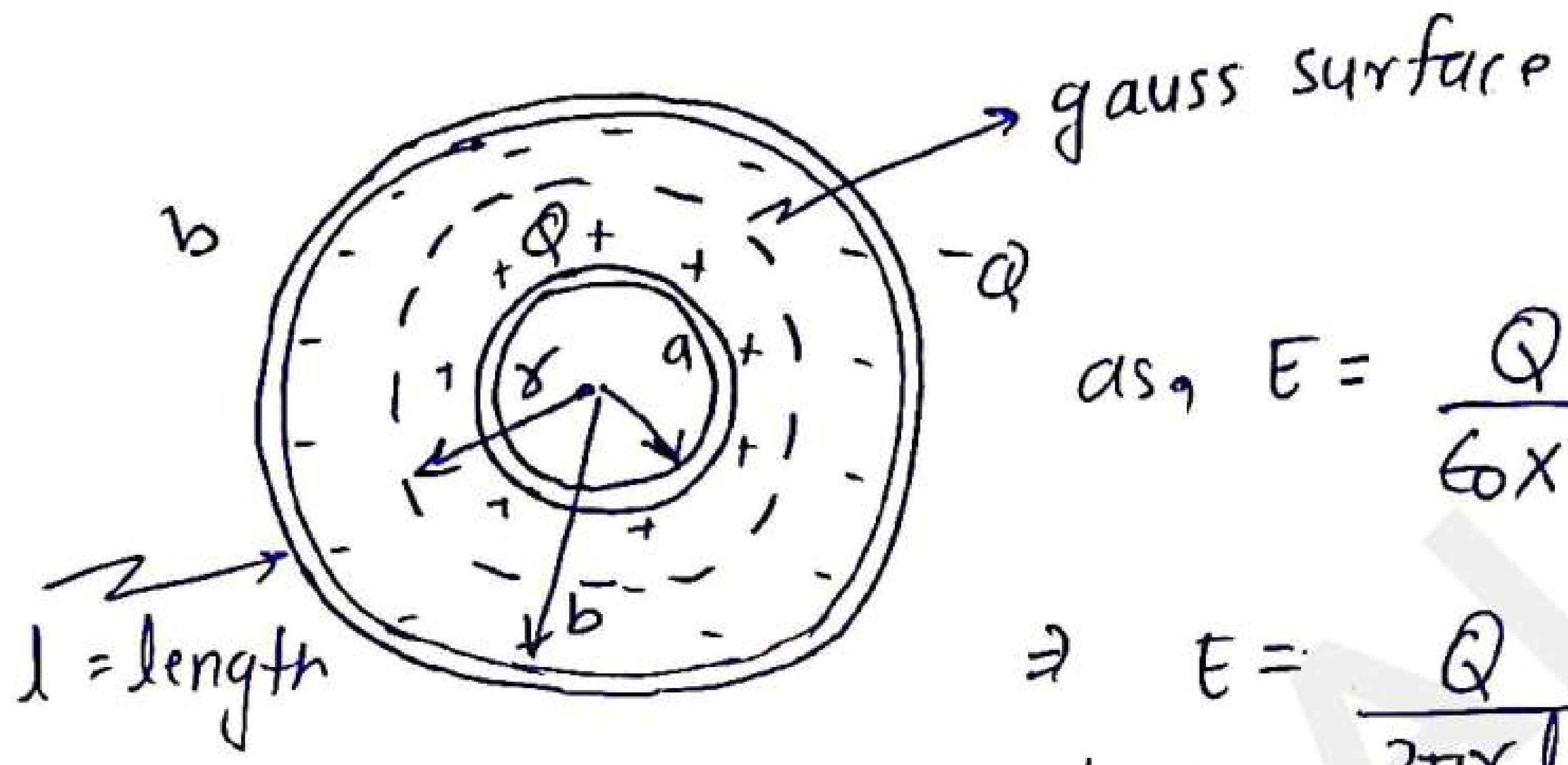
$$V = \frac{\sigma}{\epsilon_0} d \qquad \left(\sigma = \frac{Q}{A}\right)$$

$$V = \frac{Qd}{A\epsilon_0}$$

as $Q = CV$

$$\Rightarrow \boxed{C = \frac{A\epsilon_0}{d}}$$

Cylinderical Capacitor



as, $E = \frac{Q}{\epsilon_0 \times A}$ from gauss law

$\Rightarrow E = \frac{Q}{2\pi r l \epsilon_0}$

so, $dV = -\int_a^b E\,dr$

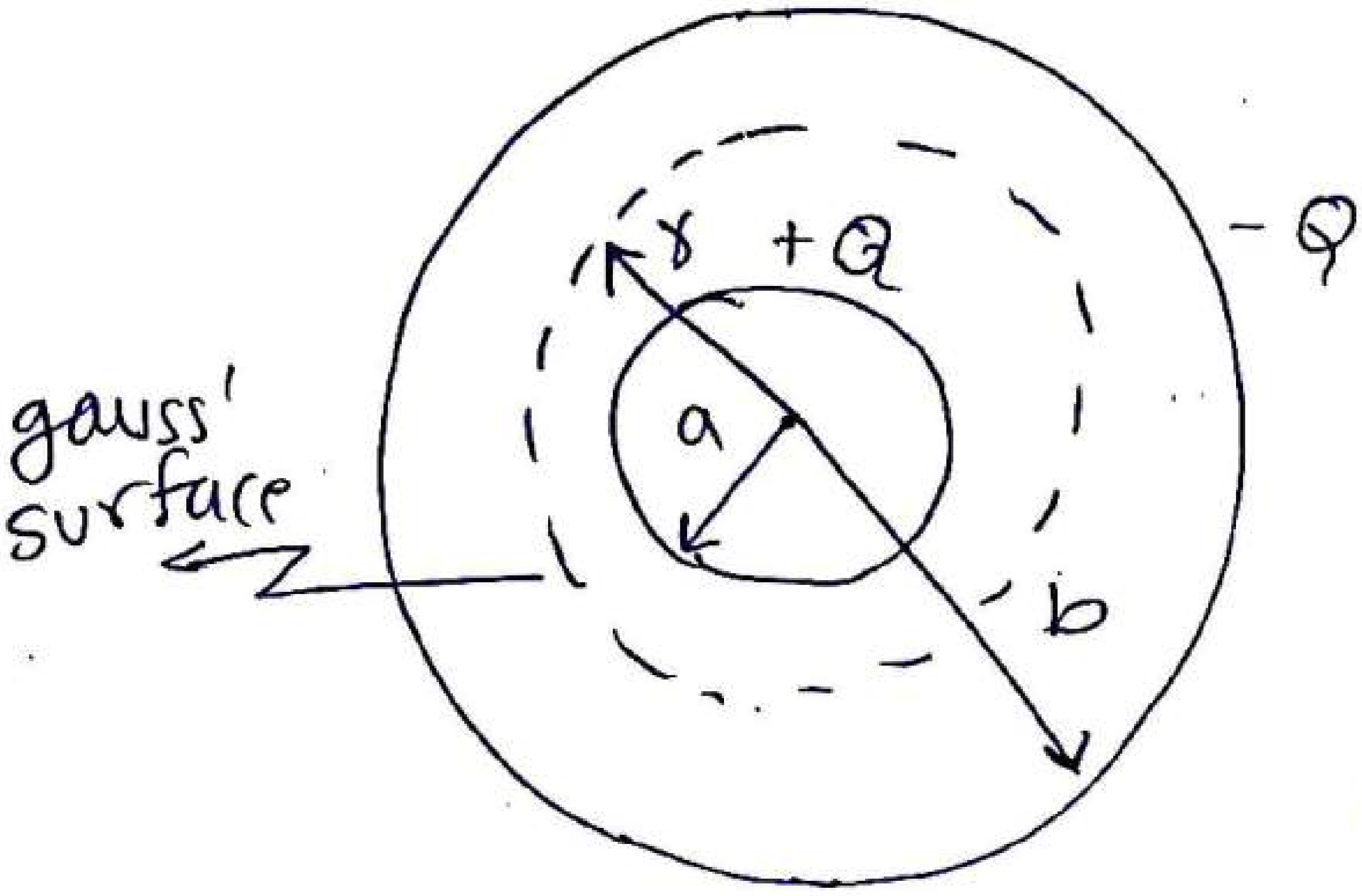
potential difference $\rightarrow \Delta V = V = -\frac{Q}{2\pi\epsilon_0 l}\int_a^b \frac{dr}{r}$

$$\boxed{V = \Delta V = \frac{Q}{2\pi\epsilon_0 l}\ln\left(\frac{b}{a}\right)}$$

as, $Q = CV$

$$\Rightarrow \boxed{C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}}$$

Spherical Capacitor:



flux $= \dfrac{Q}{\epsilon_0}$

$\Rightarrow E \times 4\pi r^2 = \dfrac{Q}{\epsilon_0}$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\int dV = -\int \vec{E}\cdot\vec{dr}$$

$$V = \Delta V = -\int_a^b \frac{Q}{4\pi\epsilon_0 r^2}\cdot dr = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_a^b$$

potential difference

$$V = \frac{Q}{4\pi\epsilon_0}\left(-\left[\frac{1}{b}-\frac{1}{a}\right]\right)$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

so, $C = \dfrac{Q}{V}$

$$\Rightarrow \boxed{C = \frac{4\pi\epsilon_0 (ab)}{(b-a)}}$$

whe $b \to \infty$

$$C = \frac{4\pi\epsilon_0 a}{\left(1-\frac{a}{b}\right)}$$

$b \to \infty$

capacitance of isolated sphere $\rightarrow \boxed{C = 4\pi\epsilon_0 a}$

Energy stored in a charged Capacitor

As energy stored → work done in storing charge Q in capacitor of capacitance C.

as $W = \int_0^Q dW = \int_0^Q (dq)V$

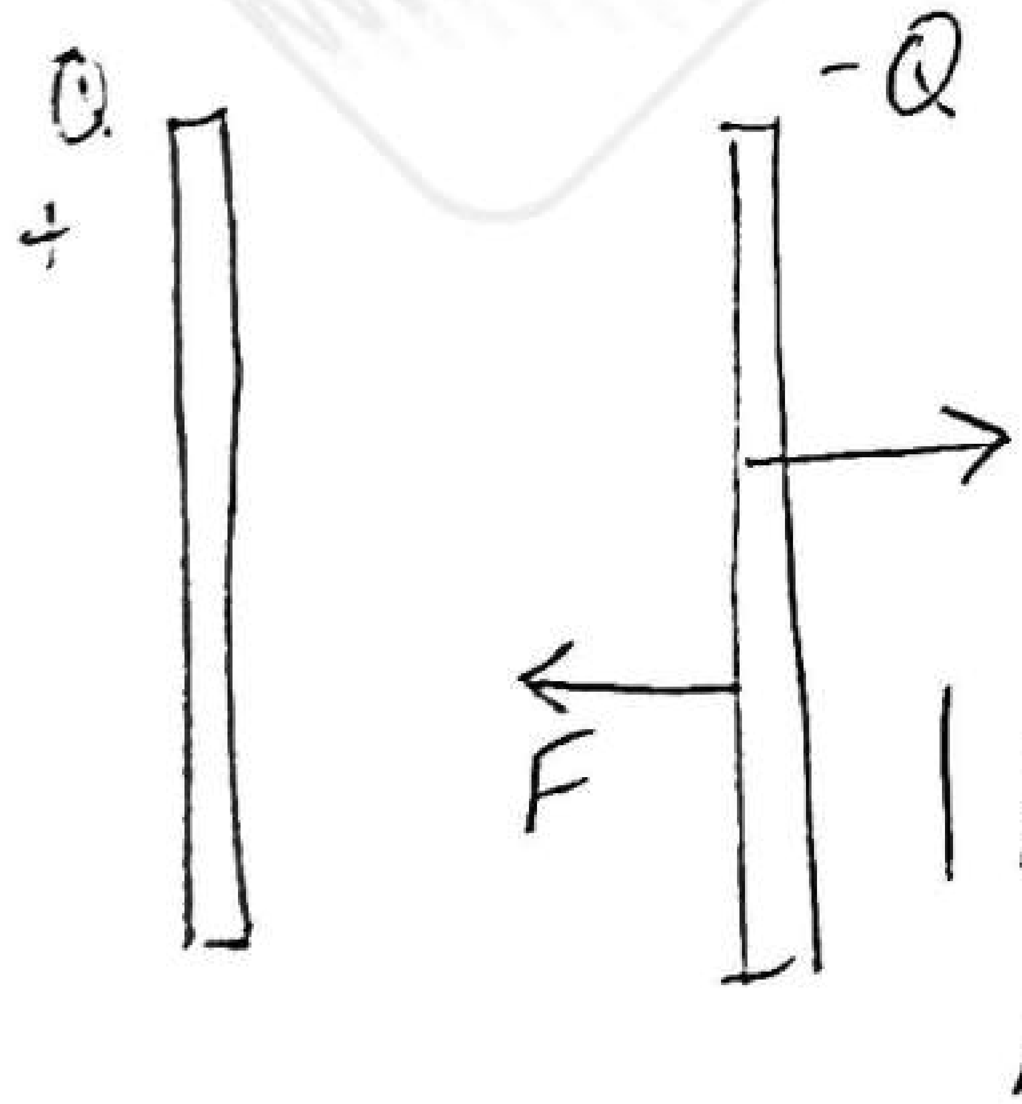
as $q = CV$

so, $V = \frac{q}{C}$

$\Rightarrow W = \frac{1}{C}\int_0^Q q\,dq$

$$\boxed{W = U = \frac{Q^2}{2C}}$$

energy stored in capacitor



↳ electric field due to plate +Q

$|F| = QE_+ = \frac{Q\sigma}{2\epsilon_0} = \frac{Q^2}{2A\epsilon_0}$

As $\boxed{\sigma = Q/A}$

Energy density → Energy / unit Volume (/ = per)

$$\frac{Q^2}{2C} \leftarrow \frac{U}{V}, \quad V \rightarrow \text{volume} = Ad$$

$$\text{energy density} = \frac{Q^2}{2\,C\,Ad}, \quad C = \frac{\epsilon_0 A}{d}$$

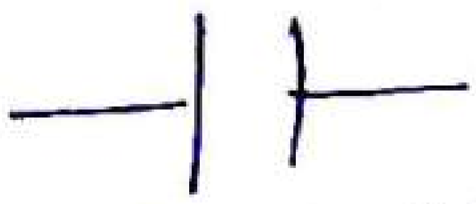
so, energy density

$$\boxed{u = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2}$$

will be same for all type of capacitor

Energy loss during Redistribution

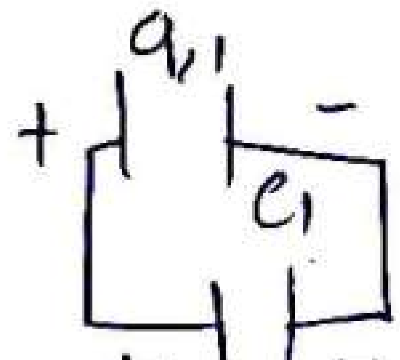
Q_1, C_1, V_1



Q_2, C_2, V_2

U_i = Initial energy

$$= \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2}$$



→ potential same across both

$$U_f = \text{final energy} = \frac{1}{2}(C_1+C_2)V^2 = \frac{1}{2}\frac{(Q_1+Q_2)^2}{(C_1+C_2)}$$

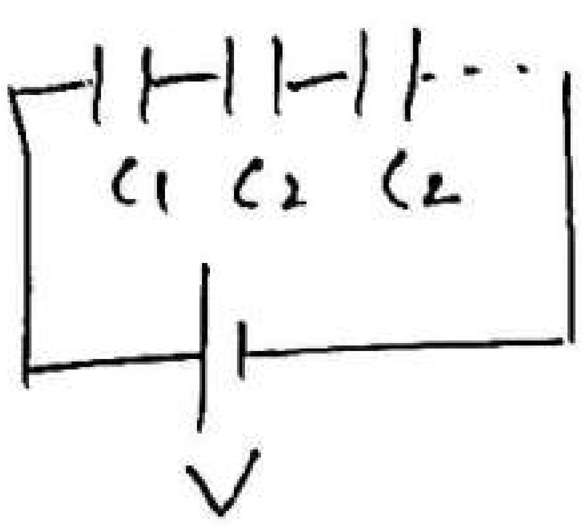
$$V = \frac{Q}{(C_1+C_2)}, \quad Q \rightarrow \text{net charge} = Q_1 + Q_2$$

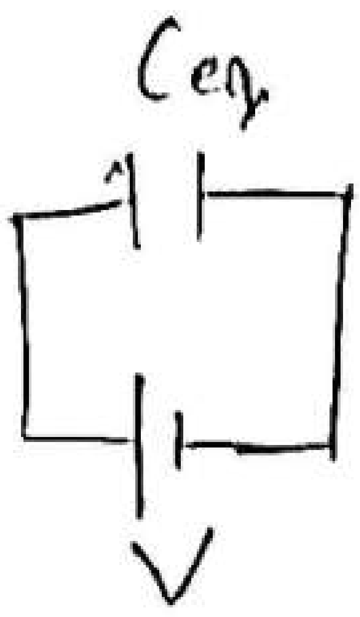
$\Delta U = \text{Energy loss} = U_i - U_f$

$$\Rightarrow \boxed{\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2}$$

COMBINATION OF CAPACITANCE:

SERIES





$V = V_1 + V_2 + \dots$

$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_2} + \dots$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots}$$

→ charge same on each capacitor

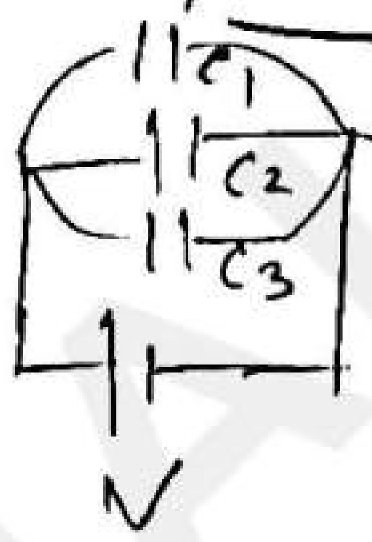
→ Energy

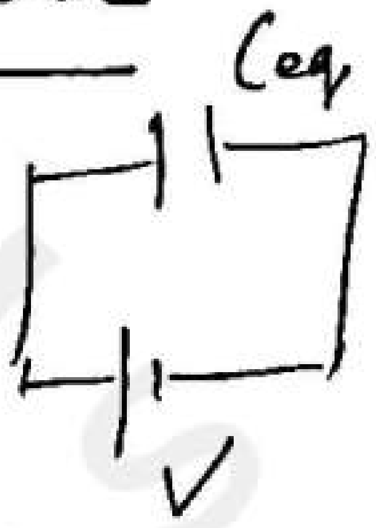
$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \dots$

$U = \frac{Q^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)$

$$\boxed{U = \frac{Q^2}{2C_{eq}}}$$

PARALLEL





$q = q_1 + q_2 + \dots$

$C_{eq}V = C_1 V + C_2 V + \dots$

$$\boxed{C_{eq} = C_1 + C_2 + \dots}$$

→ potential across each capacitor same

→ Energy:

$U = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 + \dots$

$U = \frac{1}{2}(C_1 + C_2 \dots) V^2$

$$\boxed{U = \frac{1}{2} C_{eq} V^2}$$

$V_1 = \frac{C_2 V}{C_1+C_2}$, $V_2 = \frac{C_1 V}{C_1+C_2}$ | $Q_1 = \frac{C_1 Q}{C_1+C_2}$, $Q_2 = \frac{C_2 Q}{C_1+C_2}$

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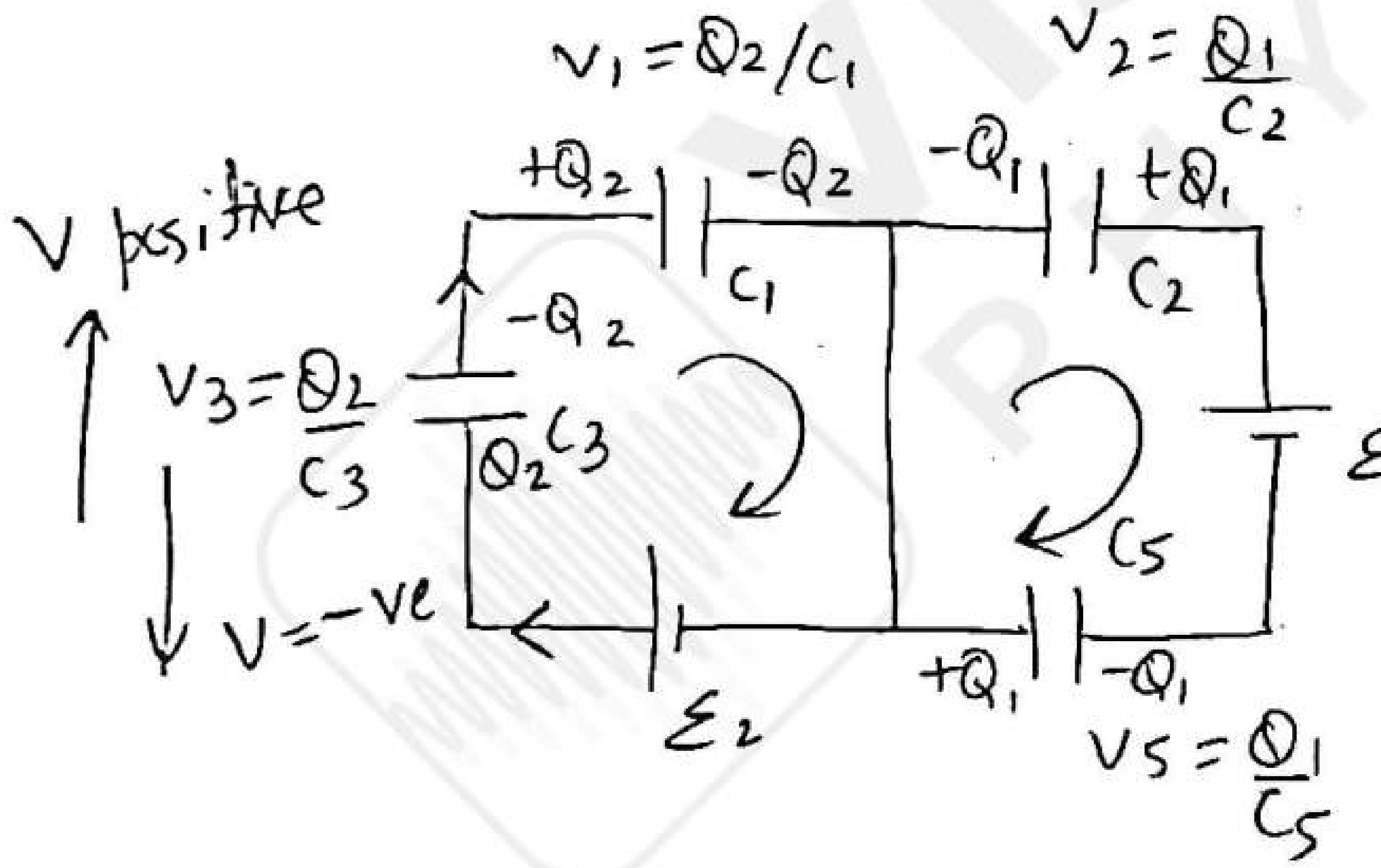
kirchhoff's rule:

→ Electric charge will remain conserved in every isolated region

→ The algebraic sum of potential changes in each closed loop must be zero.



going from (+)vely charged plate → negative plate

potential difference = -ve (vice versa)

DIELECTRIC:

→ non-conductor up to certain value of field.

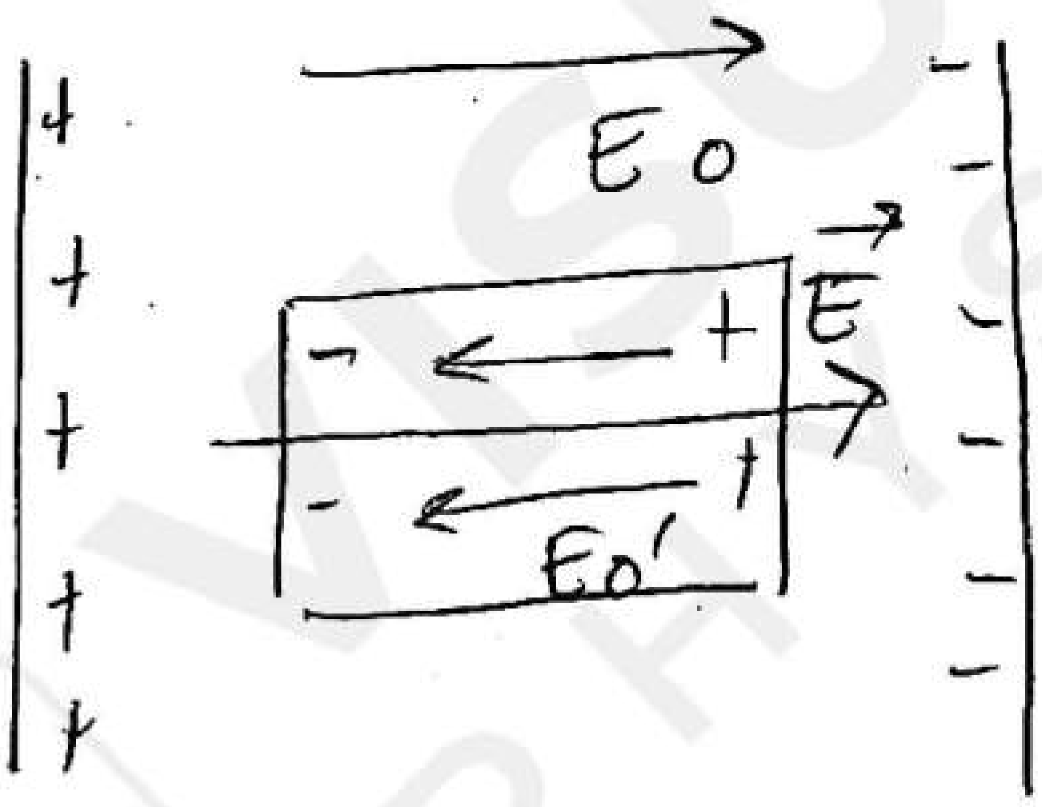
→ if field exceeds limiting value, called dielectric strength, dielectric losses insulating property and start conducting.

polar molecules

$$\boxed{K = \frac{C}{C_0}}$$

C : Capacitance with dielectric in between conductors

C_0 : Capacitance when air or vaccum is present in between plates.



so, $\boxed{\vec{E} = \frac{\vec{E_0}}{K}}$

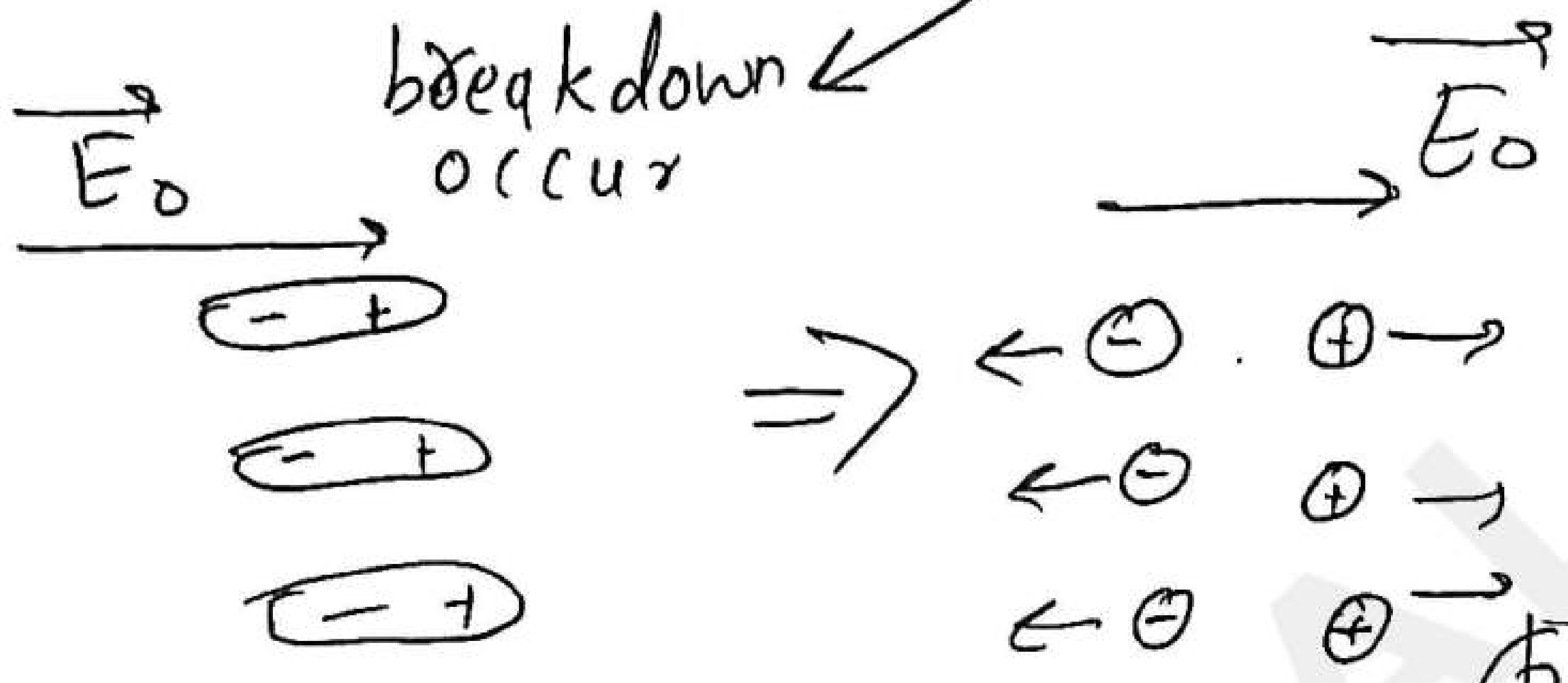
electric field reduce by 'k' factor.

polar molecules alignment.

dielectric medium

So, if dielectric material is subjected to a sufficient strong electric field

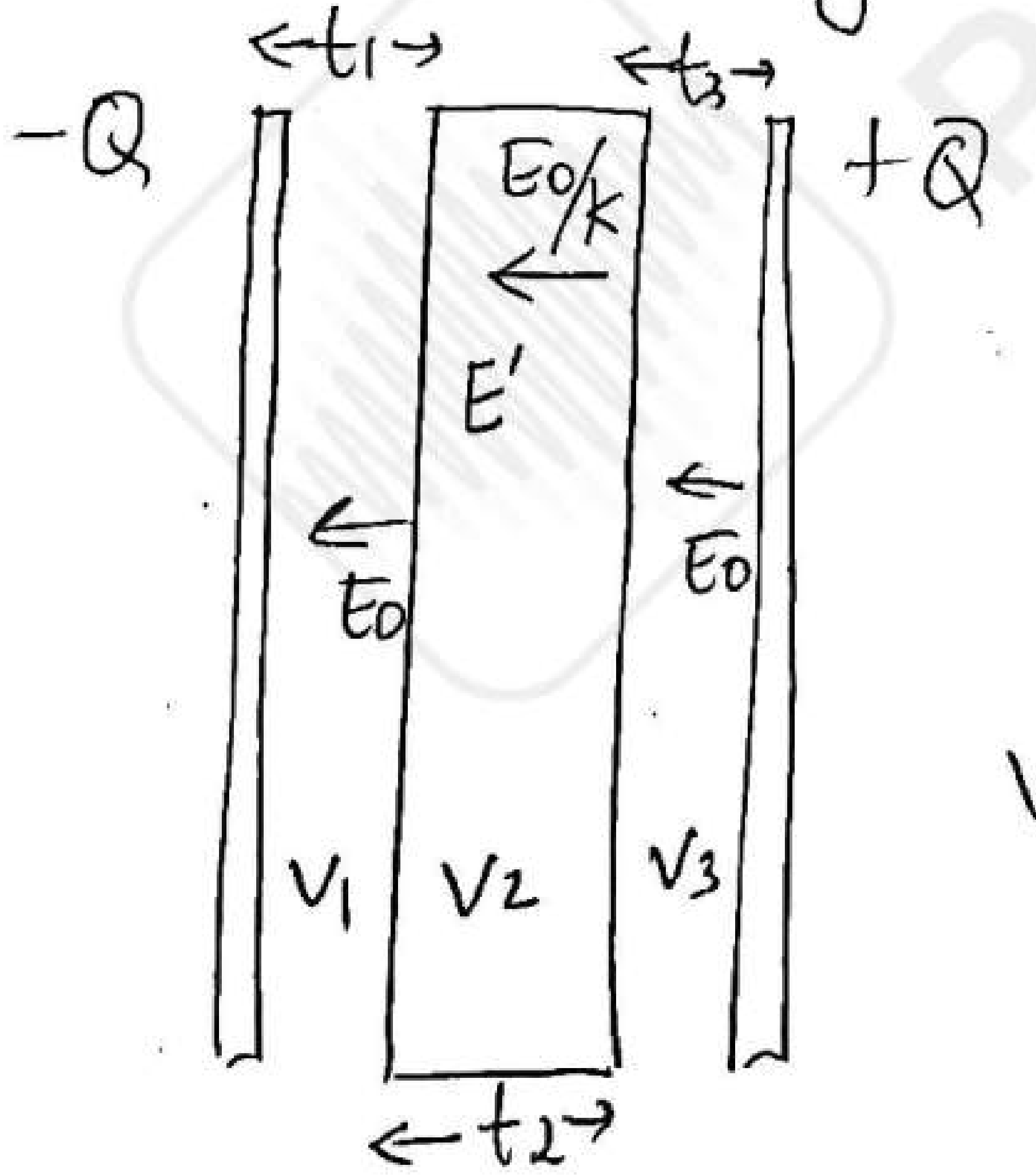
breakdown occur



hence start conduction.

⇒ Because of breakdown, capacitor have maximum potential reading

if conduction occurs by breakdown, capacitor can't store charge.



$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

$$E' = \frac{E_0}{k} = \frac{Q}{k\epsilon_0 A}$$

$$V = V_1 + V_2 + V_3$$

$$= \frac{Q}{\epsilon_0 A} t_1 + \frac{Q}{k\epsilon_0 A} t_2 + \frac{Q}{\epsilon_0 A} t_3$$

$$V = \frac{Q}{\epsilon_0 A}(t_1 + t_2) + \frac{Q}{k\epsilon_0 A} t_3$$

if $t_1+t_2+t_3 = d$

$t_3 = t$

$\Rightarrow \boxed{t_1+t_2 = (d-t)}$

so,

$$V = \frac{Q}{\epsilon_0 A}(d-t) + \frac{Q}{k\epsilon_0 A}t$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_0 A}{(d-t)+\frac{t}{k}}}$$

→ C → independent of position of slab

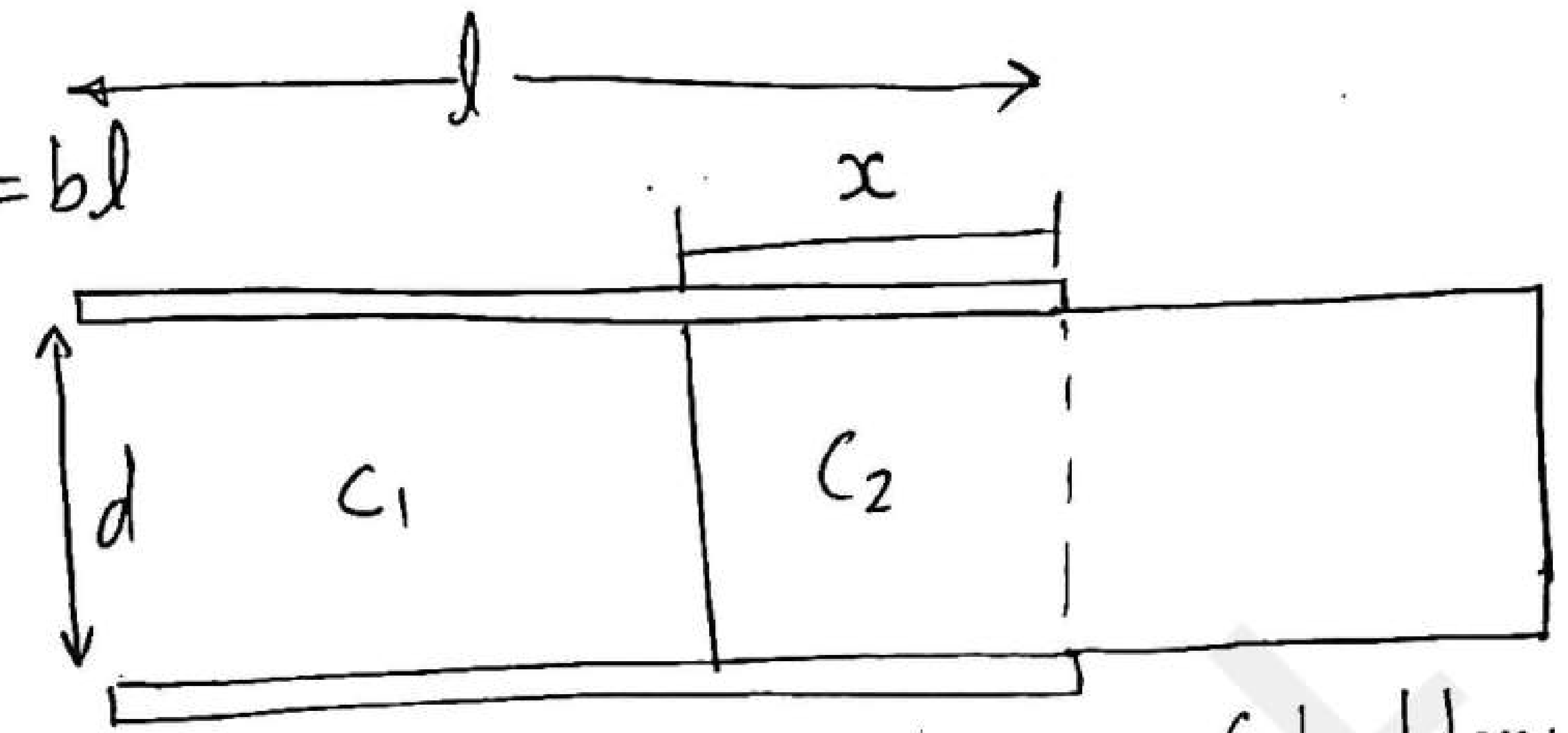
→ if $k \to \infty$, $C' = \dfrac{\epsilon_0 A}{(d-t)}$

if $t \to d$

$C' = \dfrac{k\epsilon_0 A}{d}$ $(t=d)$

Force on dielectric:

$A = bl$



(battery not connected)

$$C = \frac{\epsilon_0 A_1}{d} + k\frac{\epsilon_0 A_2}{d}$$

$$C = C_1 + C_2$$

$$A_1 = b(l-x), \quad A_2 = bx$$

$$\Rightarrow \quad C = \frac{\epsilon_0 . b}{d}[l + (k-1)x]$$

$$U(x) = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{Q^2}{2\frac{\epsilon_0 b}{d}[l+(k-1)x]}$$

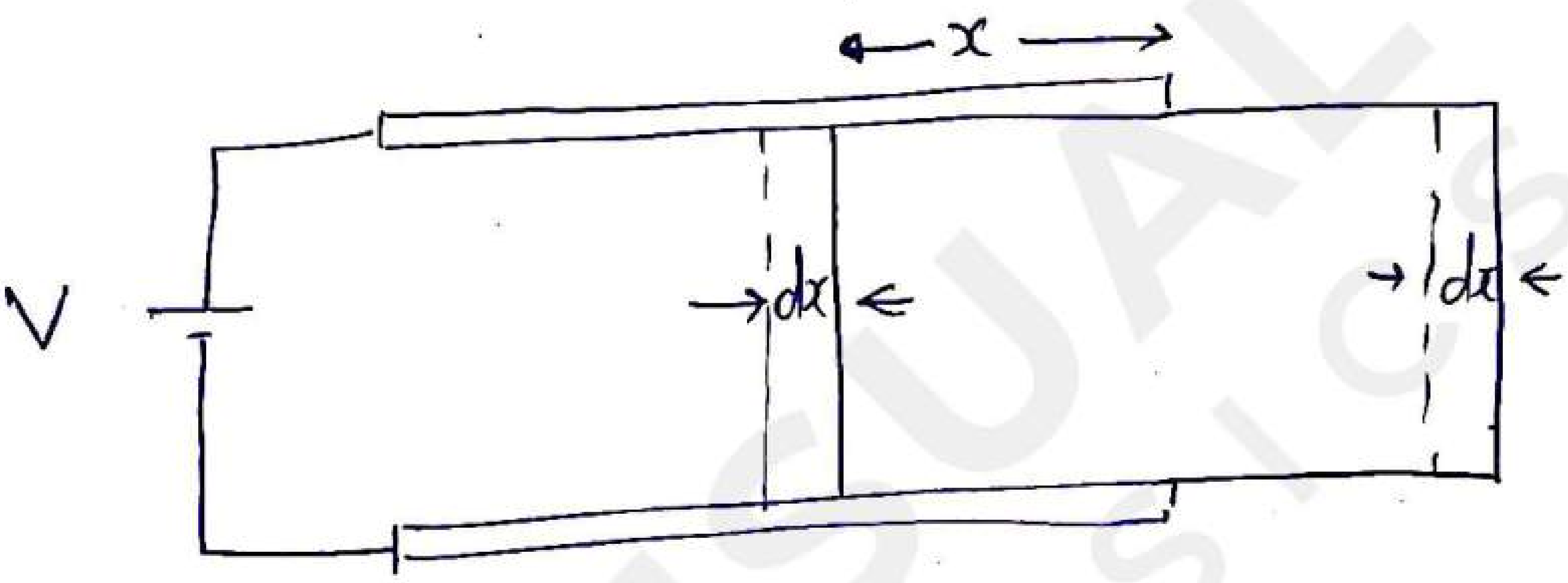
$$F(x) = -\frac{d}{dx}U(x)$$

$$\boxed{F = \frac{Q^2 d}{2\epsilon_0 b}\frac{(k-1)}{[l + x(k-1)]^2}}$$

when $x = l$

$$F(l) = \frac{Q^2 d}{2\epsilon_0 d} \frac{(k-1)}{[l + l(k-1)]^2}$$

$$F(l) = \frac{Q^2 d}{2\epsilon_0 d} \frac{(k-1)}{lK}$$



$$C = \frac{\epsilon_0 b}{d}\left(l + (k-1)x\right)$$

$$U = \frac{1}{2} C V^2 \qquad dU = \frac{1}{2}(dC)V^2$$

$$dC = \frac{\epsilon_0 b}{d}(k-1)\,dx$$

extra charge given when $C \to C + dC$

$$dQ = dC\,V = \left[\frac{\epsilon_0 b}{d}(k-1)dx\right]V$$

now $dU_{bat} = -dQ\,V = -\left[\frac{\epsilon_0 b}{d}(k-1)dx\right]V^2$

energy decreased by battery.

$$dU = \frac{1}{2}\left[\frac{\epsilon_0 b}{d}(k-1)\,dx\right]V^2$$

$$dU_{sys} = dU + dU_{bat}$$

$$dU_{sys} = -\frac{1}{2}\left[\frac{\epsilon_0 b}{d}(k-1)\,dx\right]V^2$$

as $F(x) = -\frac{d}{dx}U(x)$

$$\Rightarrow \boxed{F(x) = \frac{1}{2}\left[\frac{\epsilon_0 b}{d}(k-1)\right]V^2}$$