VISUAL PHYSICS

SHORT NOTES

CHAPTER

Electric Power

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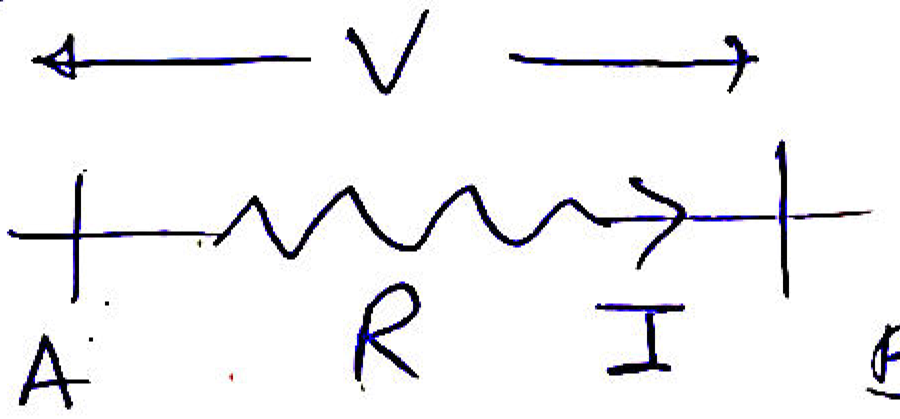
ELECTRIC POWER

heating effect → when current pass through a conductor it becomes hot.

due to collisions of drifting electrons inside the conductor, loss of electrons K.E results in heating of conductor.

potential difference



amount of charged pass in 't' seconds

$q = i t$

loss in potential energy $= qV$

$= iVt$

as $V = iR$

$\Rightarrow$ $\boxed{\text{Energy loss} = i^2 R t}$

or Energy loss $= \frac{V^2}{R} t$

So in terms of energy loss per seconds

i.e. $\boxed{\text{power loss} = i^2 R = \frac{V^2}{R}}$

Joules law of heating,

Heat produced $\propto i^2$

$\propto t$

$\propto R$

$\Rightarrow$ heat lost $= i^2 R t$

$\boxed{1\,cal = 4.18\,J = 4.2\,J}$

$$\boxed{\begin{aligned} 1\,kWh &= 1000\,W \times hour = 1000\,W \times 3600\,s \\ &= 3.6 \times 10^6\,J \end{aligned}}$$

$\boxed{1\,kWh \rightarrow \text{power consumed } 1\,kW \text{ for one hour}}$

$\boxed{1\,hp = 746\,W}$

→ In series ⟶ as current is same in series

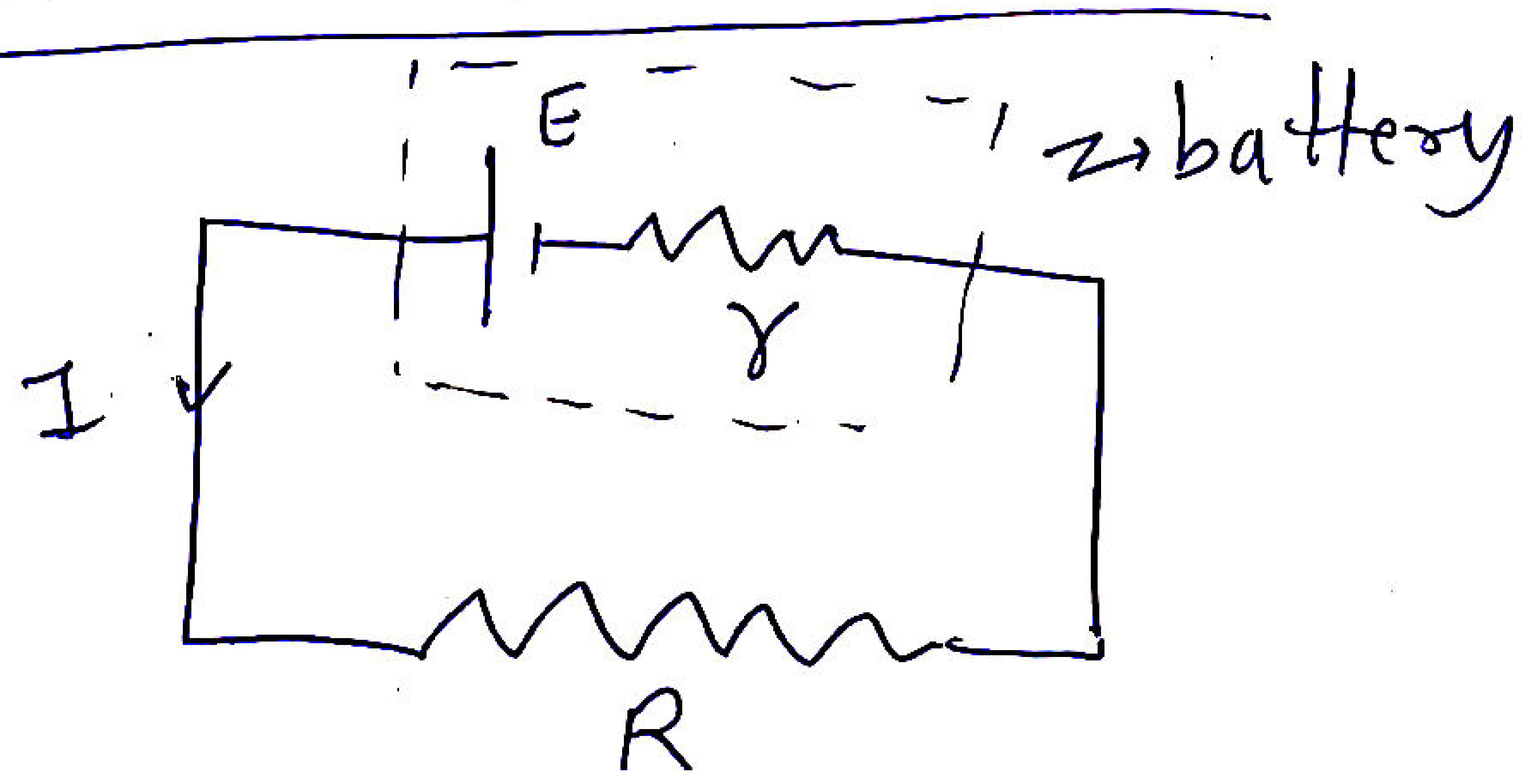
$P = I^2 R$, more R more Power loss

→ In parallel:

$P = \dfrac{V^2}{R}$, less R, more power loss.

as resistor have Same potential difference in parallel.

Maximum power Transfer



As power loss in external resistor $= I^2 R$

$$I = \frac{E}{R+r}$$ $E \rightarrow$ emf

$$P = \frac{E^2 R}{(R+r)^2}$$

$$\frac{dP}{dR} = 0, \quad E^2\left[\frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4}\right] = 0$$

$$\Rightarrow \quad (r+R) = 2R \quad \text{or} \quad \boxed{r = R}$$

So if external resistance = internal resistance than maximum power output to external system R.

Electric Appliances :

Each Appliance requires power

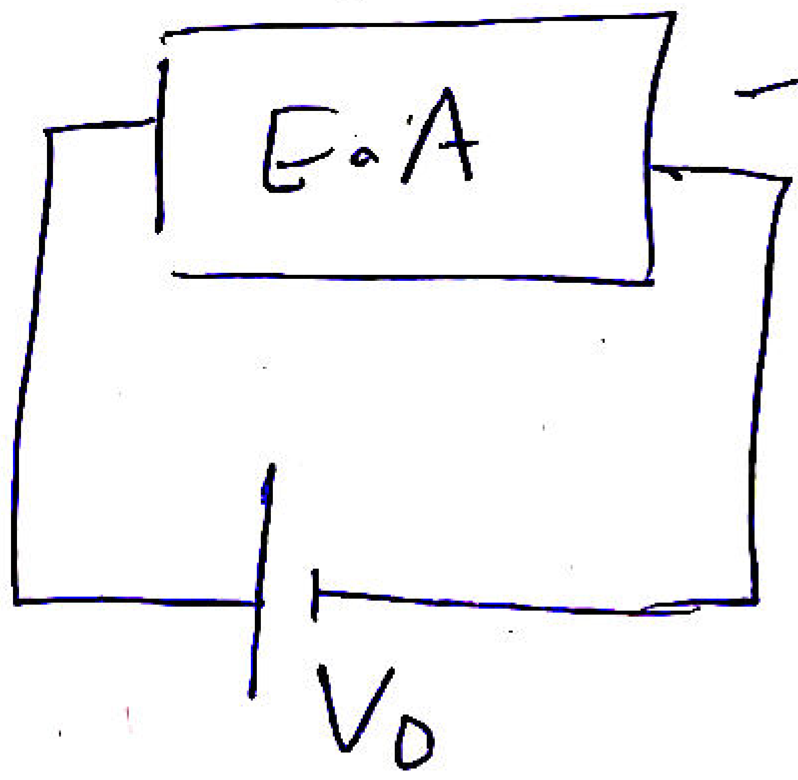
$\hookrightarrow$ Rating $[P_0, V_0]$

$P_0 \rightarrow$ Rated power

$V_0 \hookrightarrow$ Rated voltage

that means:

Electric Appliance



$\rightarrow P_0$

$$\Rightarrow P_0 = \frac{V_0^2}{R}$$

or
$$\boxed{R = \frac{V_0^2}{P_0}}$$

if power supplied by V

$$\Rightarrow P = \frac{V^2}{R} = \frac{V^2}{V_0^2} P_0$$

$$\Rightarrow \boxed{P = \left(\frac{V}{V_0}\right)^2 P_0}$$

as appliance work on specific maximum current.

if $\boxed{V > V_0 \Rightarrow P > P_0} \rightarrow$ damage the appliance.

$$\boxed{V < V_0 \Rightarrow P < P_0}$$

Long distance power transmission:

Power loss $= I^2R = P$

also $P_0' = VI$

$\hookrightarrow$ given power

$\Rightarrow I = \frac{P_0'}{V}$

$\Rightarrow$ Power loss $= \frac{P_0'^2}{V^2} R$

$\Rightarrow$ power loss $\propto \frac{1}{V^2}$

more the potential drop, less the power loss.

OR say less the current, less the power loss. So for given power P_0', V should be high, I should be low

$\Rightarrow$ so transmission lines are high voltage power supplie

Appliance Combination

P_{01} $P_{0,2}$

V_0 V_0

$P_{01} > P_{0,2}$

$\rightarrow \frac{V_0^2}{R_1} > \frac{V_0^2}{R_2}$

$\boxed{R_1 < R_2}$

higher power rating for same V_0, means less Resistance.

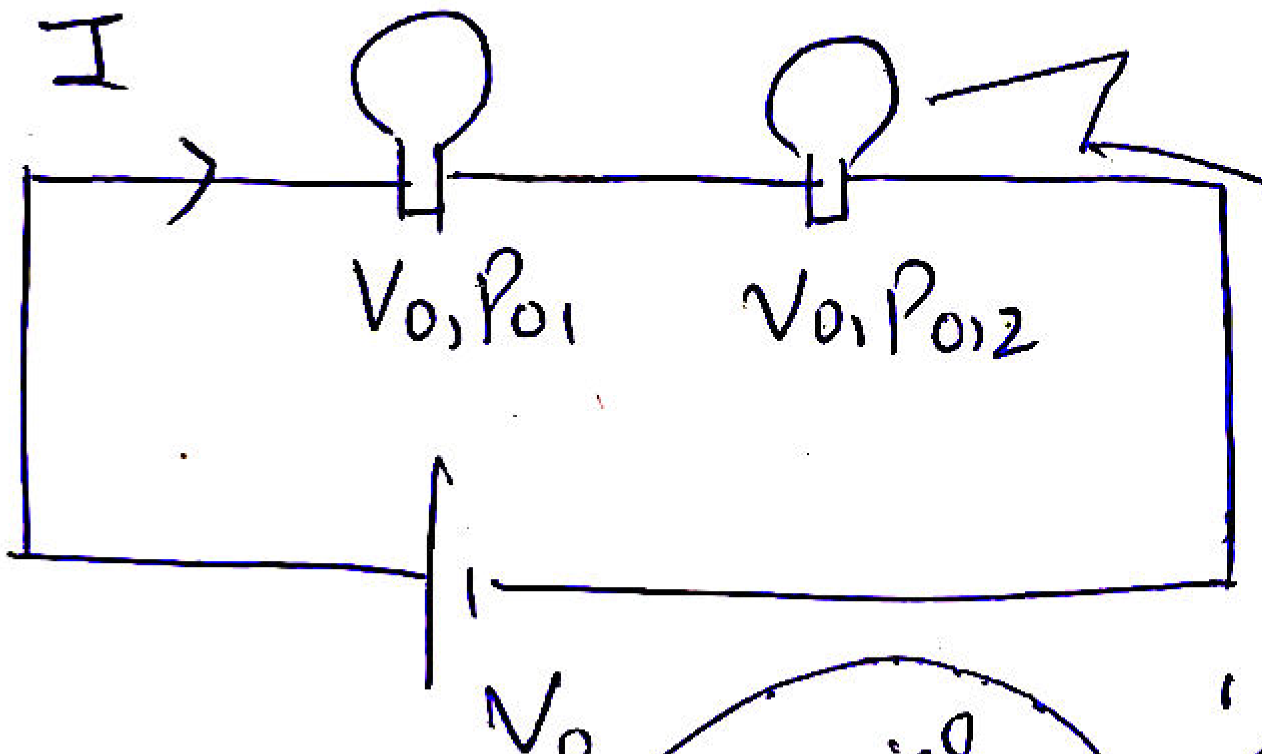
$R_1 < R_2$

I is same

so, $P_1 : P_2 = R_1 : R_2$

$\boxed{P_1 < P_2}$

I



hence,

that means bulb 2 glow brighter

In series low power rating glow brighter

$I = \frac{V_0}{R_1 + R_2}$

$P_1 = I^2 R_1$

$P_2 = I^2 R_2$

$\boxed{P_{net} = I^2(R_1 + R_2)} \boxed{= P}$ total power consumed

$$P = \frac{V_0^2}{(R_1+R_2)^2}(R_1+R_2)$$

$$P = \frac{V_0^2}{(R_1+R_2)}$$

or $$\frac{1}{P} = \frac{R_1+R_2}{V_0^2} = \frac{R_1}{V_0^2} + \frac{R_2}{V_0^2}$$

$$\frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}}$$

or $$\boxed{P = \frac{P_{01} \times P_{02}}{P_{01}+P_{02}}}$$

only when applied voltage $V = V_0$

if $V \neq V_0$

$$P' = \left(\frac{V}{V_0}\right)^2 P$$

$$\boxed{P' = \left(\frac{V}{V_0}\right)^2 \left[\frac{P_{01} \times P_{02}}{P_{01}+P_{02}}\right]}$$

or In general

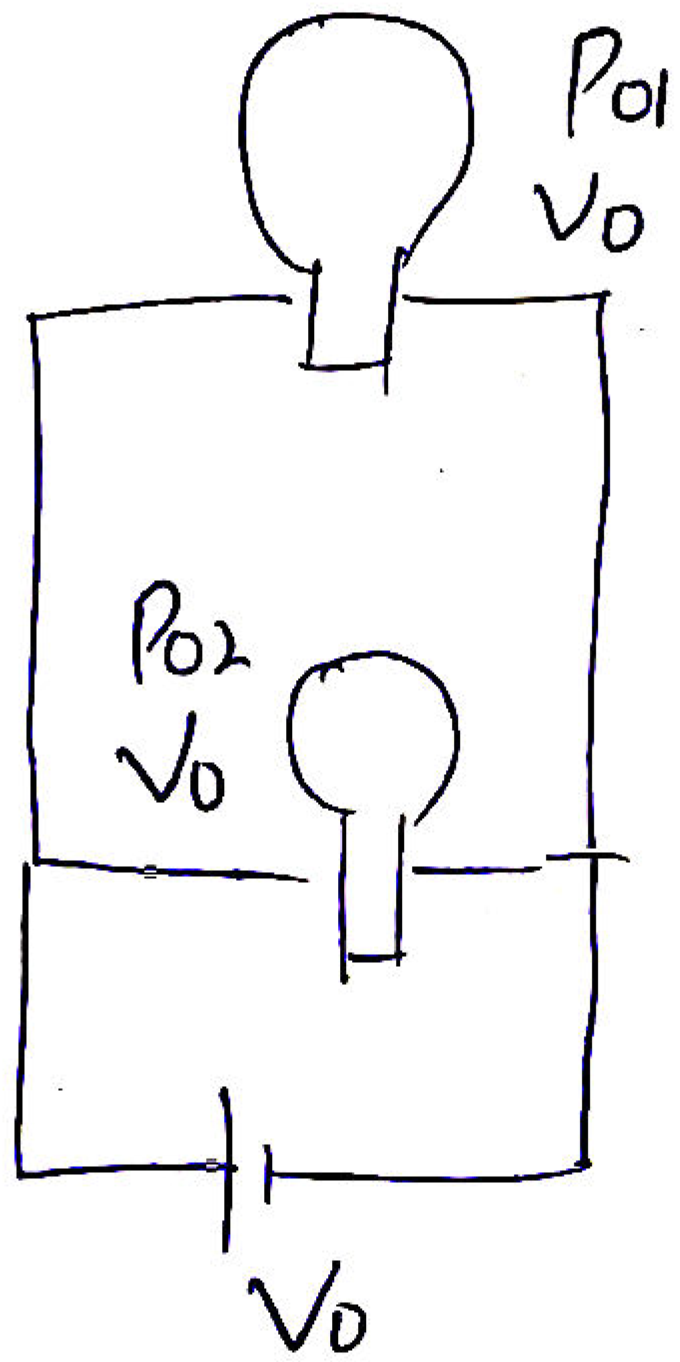
$$\boxed{\frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}} + \dots \frac{1}{P_{0n}}}$$

Power consumed when $V = V_0$

$V_0 \rightarrow$ Rated voltage

$V \rightarrow$ applied voltage

$P_{01} > P_{02}$



$R_1 < R_2$ ⇝ for parallel

$$P_1 : P_2 = \frac{V^2}{R} : \frac{V^2}{R_2}$$

$$P_1 : P_2 = \frac{1}{R_1} : \frac{1}{R_2}$$

$\Rightarrow$ $P_1 > P_2$

so 1 glow brighter in parallel

$$P_1 = \frac{V_0^2}{R_1} = P_{01}$$

$$P_2 = \frac{V_0^2}{R_2} = P_{02}$$

$$\boxed{P = P_1 + P_2 = P_{01} + P_{02}}$$

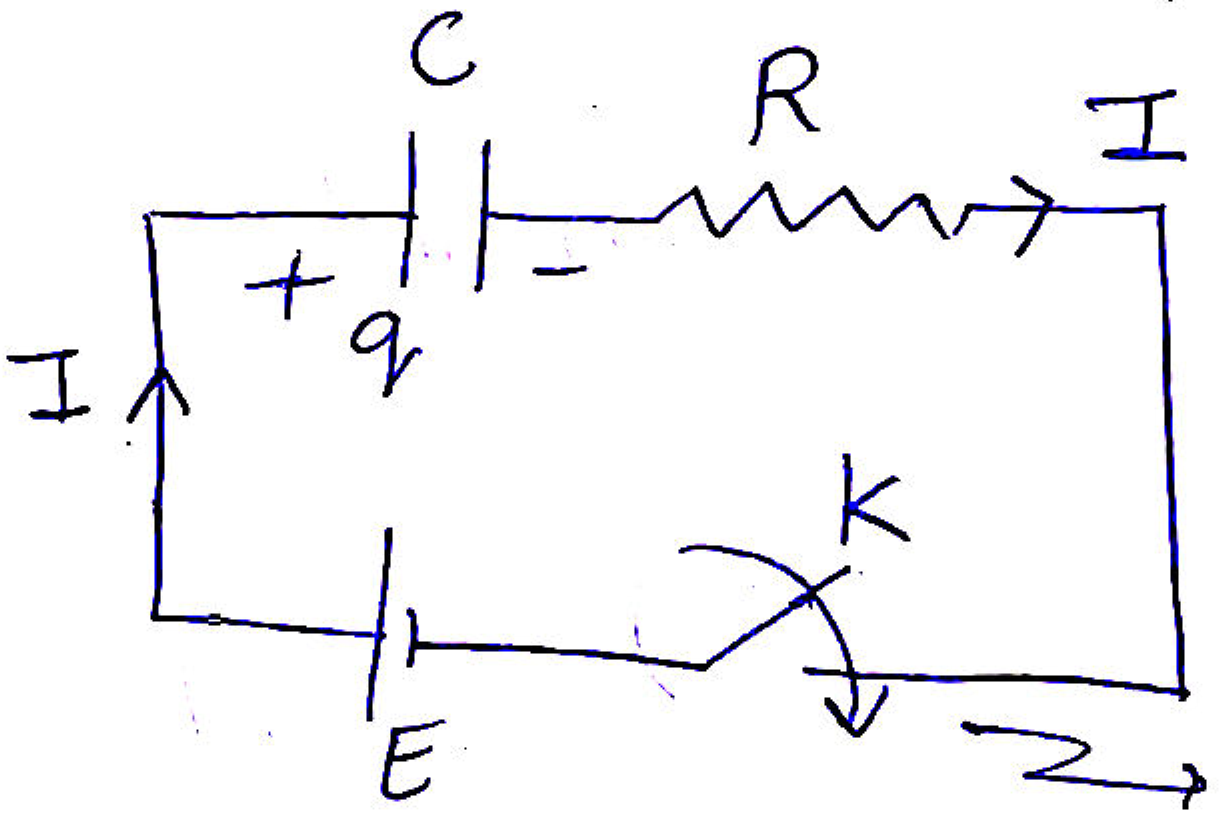
$\rightarrow$ if $V = V_0$

if $V \neq V_0$

$$P' = \left(\frac{V}{V_0}\right)^2 P = \left(\frac{V}{V_0}\right)^2 [P_{01} + P_{02}]$$

in general $\Rightarrow$ $\boxed{P = P_1 + P_2 + \ldots P_n}$

Charging of Capacitor:



initial conditions:

$t=0,\ q=0$

$I = \frac{dq}{dt}$

$\rightarrow$ key is closed

Applying kirchhoff's law

$$E = \frac{q}{C} + IR$$

$$E = \frac{q}{C} + \frac{dq}{dt}R$$

$$\Rightarrow \int R\frac{dq}{dt} = \int -\frac{q}{C} + E$$

$$\Rightarrow \int_0^{q_0} \frac{dq}{(EC-q)} = \int_0^t \frac{dt}{RC}$$

$$\ln(EC-q) = \ln EC - \frac{t}{RC}$$

$$\ln\left[\frac{EC-q}{EC}\right] = -\frac{t}{RC}$$

$$\frac{EC-q}{EC} = e^{-t/RC}$$

$$\boxed{q = EC\left[1-e^{-t/RC}\right]}$$

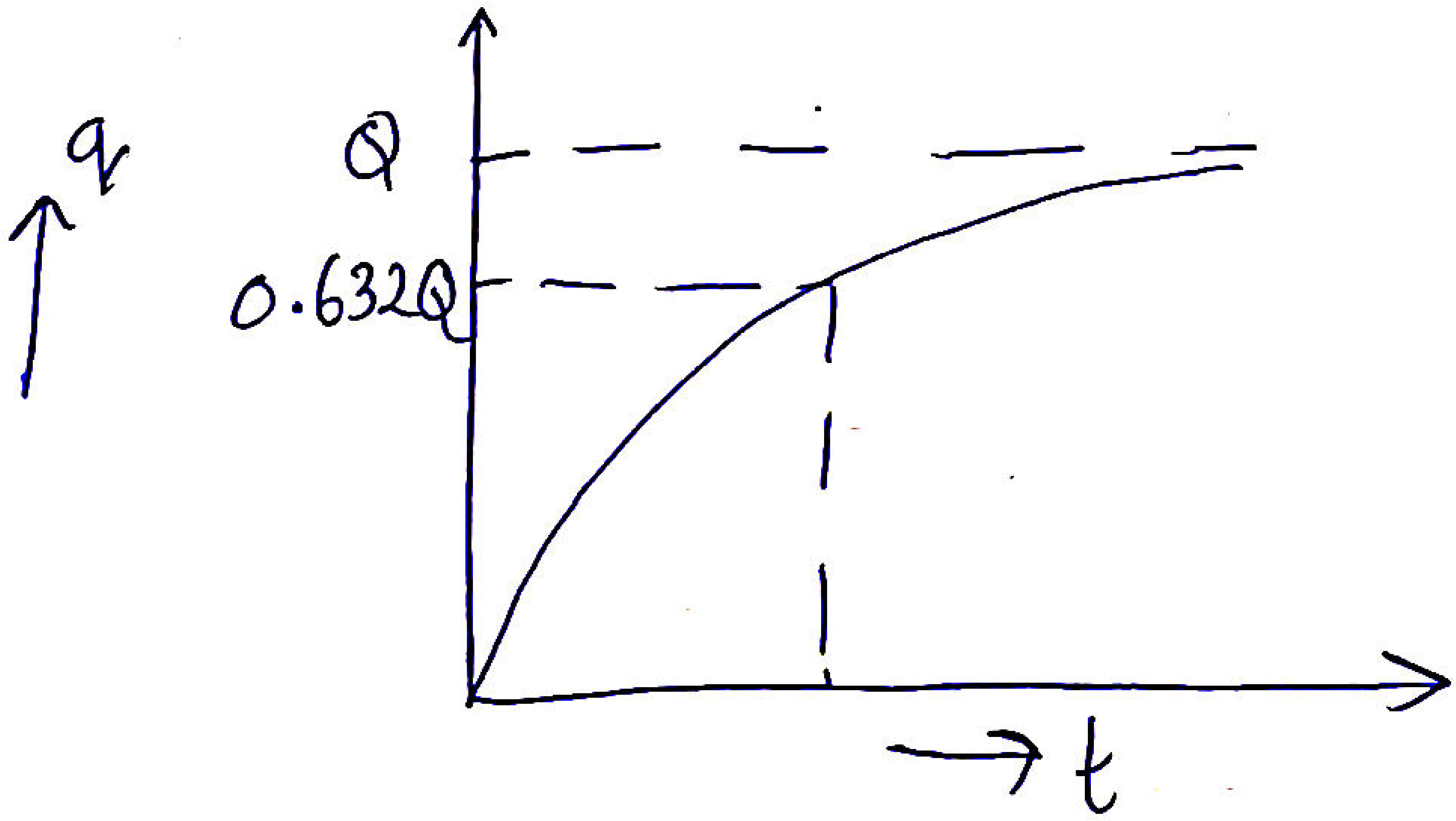
$\tau = RC \rightarrow$ time constant

Whe $t = \tau$

$$q = Q\left(1-e^{-\tau/\tau}\right) = Q\left(1-e^{-1}\right)$$

$$\boxed{q = 0.632Q}$$

So, $t \rightarrow \tau$, charging to 63.2%

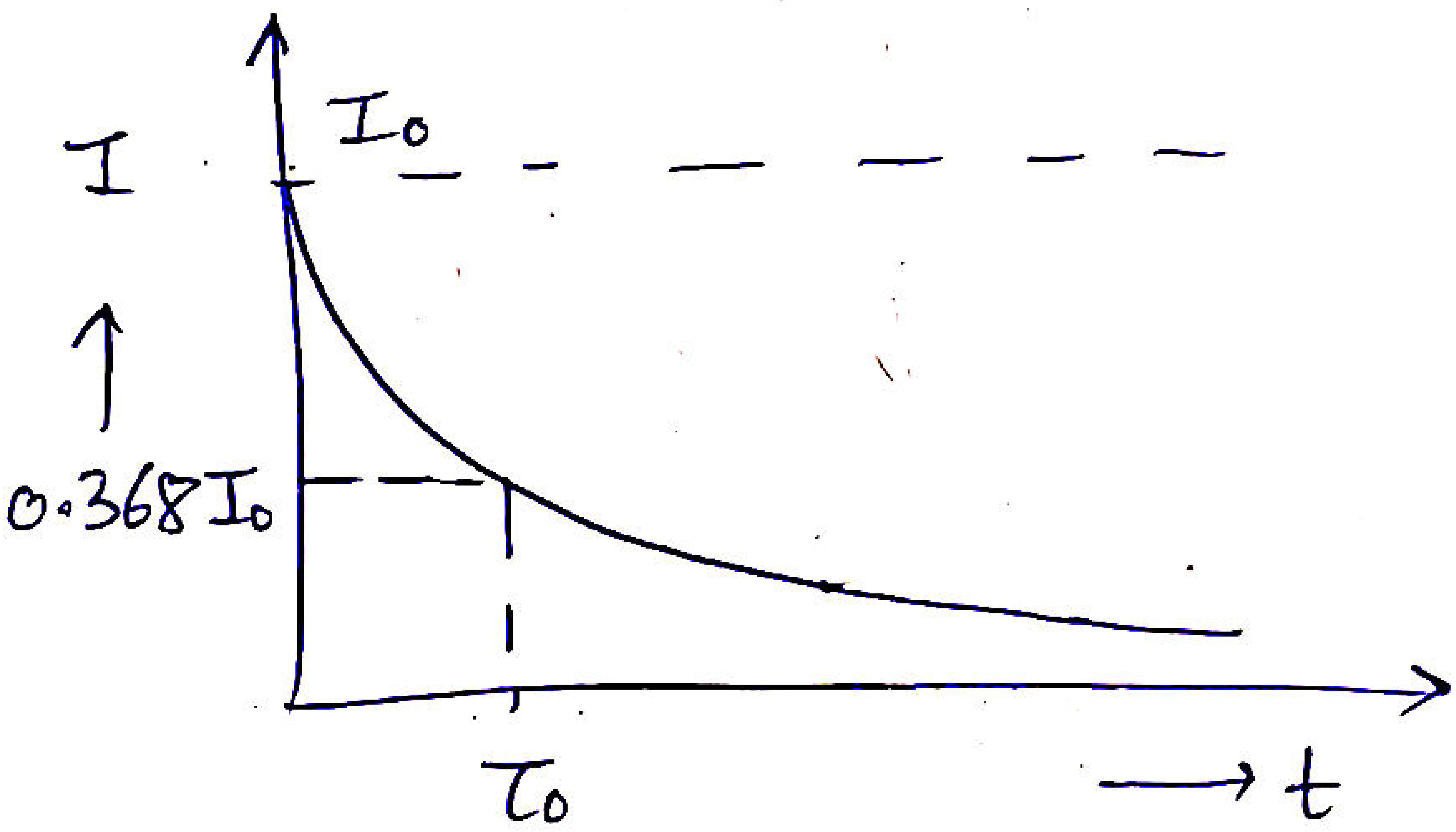


$$I = \frac{dq}{dt} = Q\left[0 - e^{-t/\tau}\left(-\frac{1}{\tau}\right)\right]$$

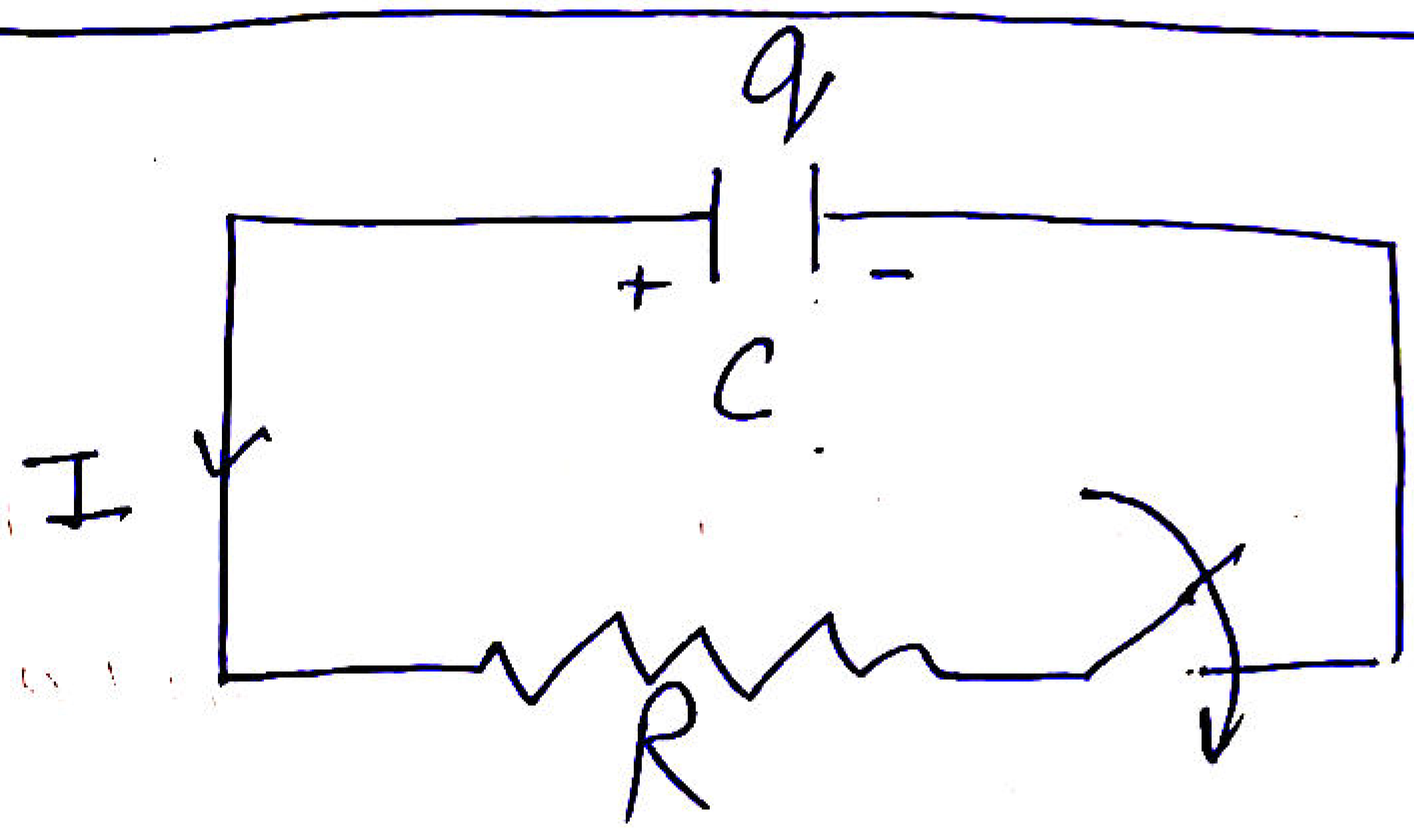
$$I = \frac{Q}{\tau} e^{-t/\tau} \Rightarrow \boxed{I = \frac{EC}{RC} e^{-t/\tau}}$$

$$\boxed{I = I_0 e^{-t/\tau}}$$

When $t \to \tau$ $\Rightarrow$ $\boxed{I = \frac{I_0}{e} = 0.368 I_0}$



DISCHARGING OF CAPACITOR:



At $t=0$,
$q = Q \Rightarrow$ initial charge

$$I = \frac{-dQ}{dt}$$

Applying kirchhoff's law:

$$\frac{q}{C} = IR \Rightarrow \frac{q}{C} = -\frac{dq}{dt} R$$

$$\int_{Q}^{q} \frac{dq}{q} = -\int_{0}^{t} \frac{dt}{RC}$$

$$\Rightarrow \ln\left(\frac{q}{Q}\right) = -t/RC$$

$$\Rightarrow \boxed{q = Q e^{-t/RC}}$$

$RC = \tau$

$$\Rightarrow \boxed{q = Q e^{-t/\tau}}$$

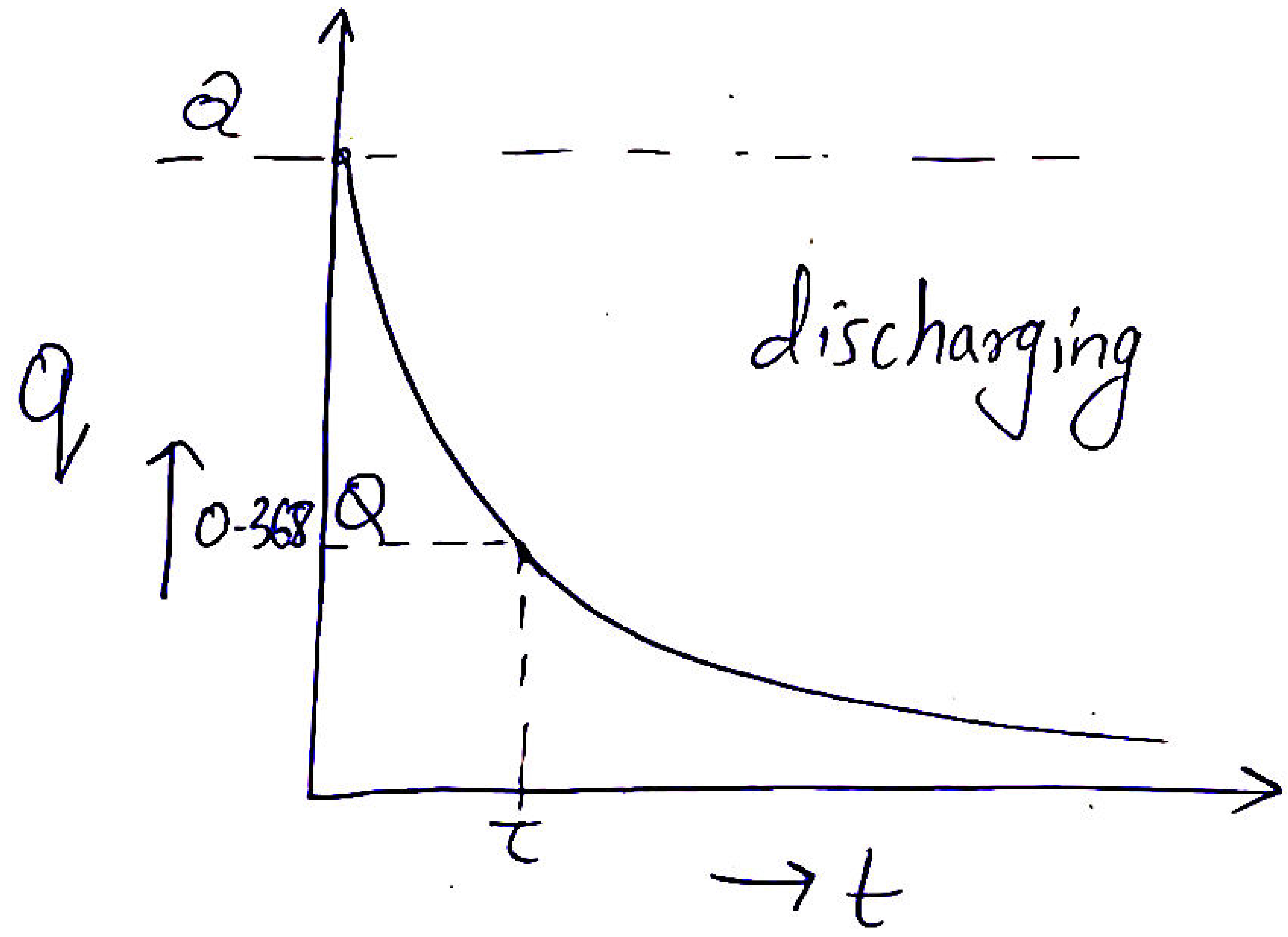
when $\boxed{t = \tau}$

$$\Rightarrow q = Q e^{-\tau/\tau}$$

$$\boxed{q = Q e^{-1} = \frac{Q}{e}}$$

$$\Rightarrow \boxed{q = 0.368 Q}$$

so $t \rightarrow \tau$, $q \rightarrow 36.8\%$ of initial charge



Energy Considerations in charging :

Total work done by battery in fully charging the capacitor

$$W_b = EQ = E^2C \quad [\text{as } Q = EC]$$

E → emf of battery

$$\boxed{U = \frac{1}{2}CE^2}$$

as $\cancel{J}$ $H = \int_0^\infty I^2 R\,dt = \int_0^\infty I_0^2 e^{-2t/\tau} R\,dt$

H → Heat dissipated in resistor

$$= I_0^2 R \left[\frac{e^{-2t/\tau}}{-2\tau}\right]_0^\infty$$

$$H = \frac{-I_0^2 R\tau}{2}(0-1)$$

so, total $\boxed{W_b = CE^2}$ one half → dissipated in heat $\boxed{H = \frac{1}{2}CE^2}$

Energy Consideration in discharging:

energy loss, $H = \int_0^\infty I^2 R\, dt$

$$\Rightarrow H = \int_0^\infty \left(\frac{Q}{RC}\right)^2 e^{-2t/\tau} R\, dt$$

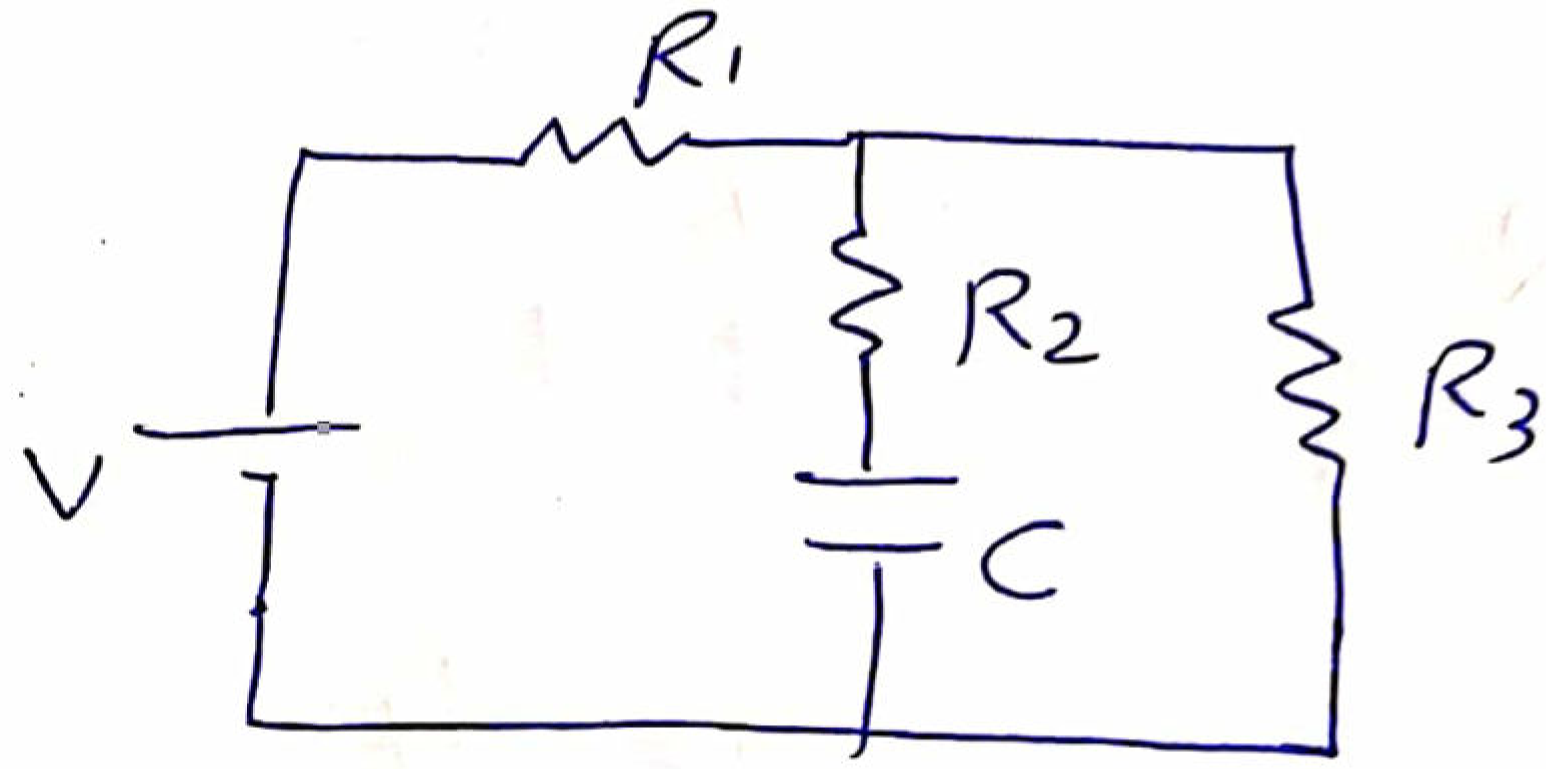
$$H = \frac{-Q^2 RC}{2RC^2}[0-1]$$

$$\boxed{H = \frac{1}{2}\frac{Q^2}{C}}$$

initial energy stored in capacitor

So whole energy is dissipated in form of heat.

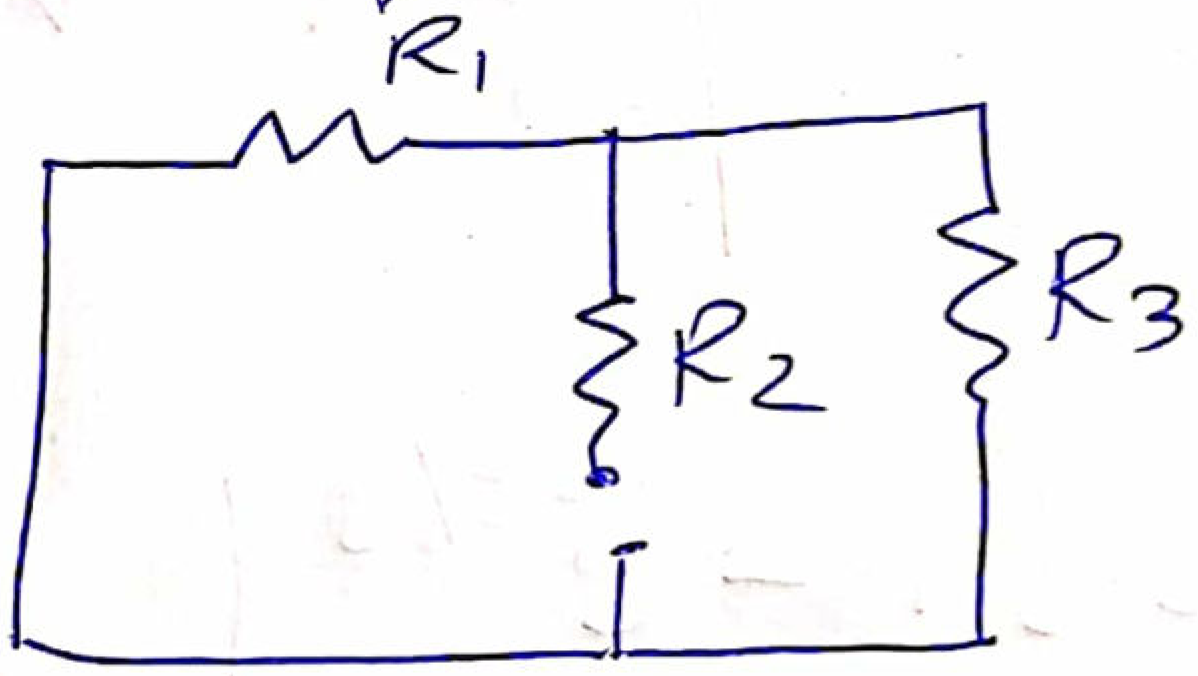
Equivalent Time Constant



$\tau_{eq} = (R_{eq})\,C$

replace battery by short-circuit and capacitor by open circuit and find R_{eq} around C

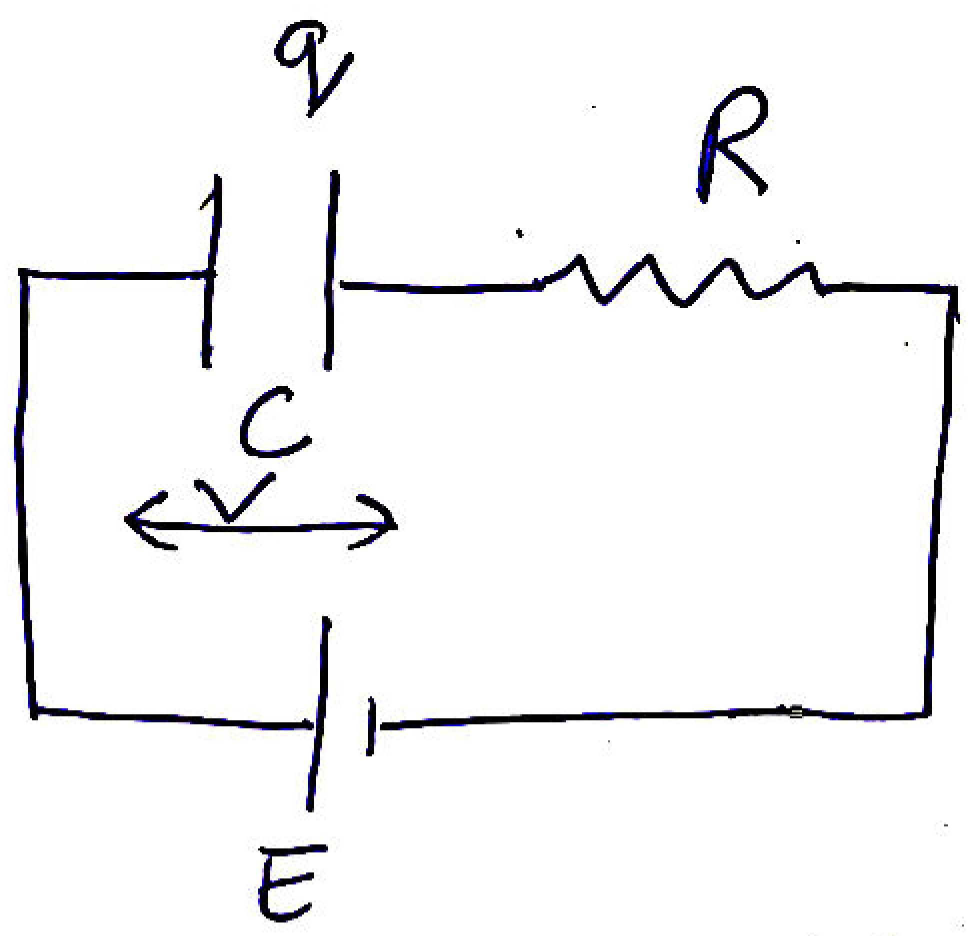
$\Rightarrow$



$\Rightarrow$ $\boxed{R_{eq} = \frac{R_1 \cdot R_3}{R_1 + R_3} + R_2}$

So $\boxed{\tau_{eq} = R_{eq}\,C}$

for charging RC circuit:



$$V = \frac{q}{C}$$

$$P_C = \frac{dU_C}{dt}$$

Power transferred to capacitor

$$q = Q\left[1 - e^{-t/\tau}\right]$$

$$Q = EC$$

$$\Rightarrow i = \frac{E}{R} e^{-t/\tau}$$

$$\Rightarrow P_c = Vi = \frac{q}{C} i$$

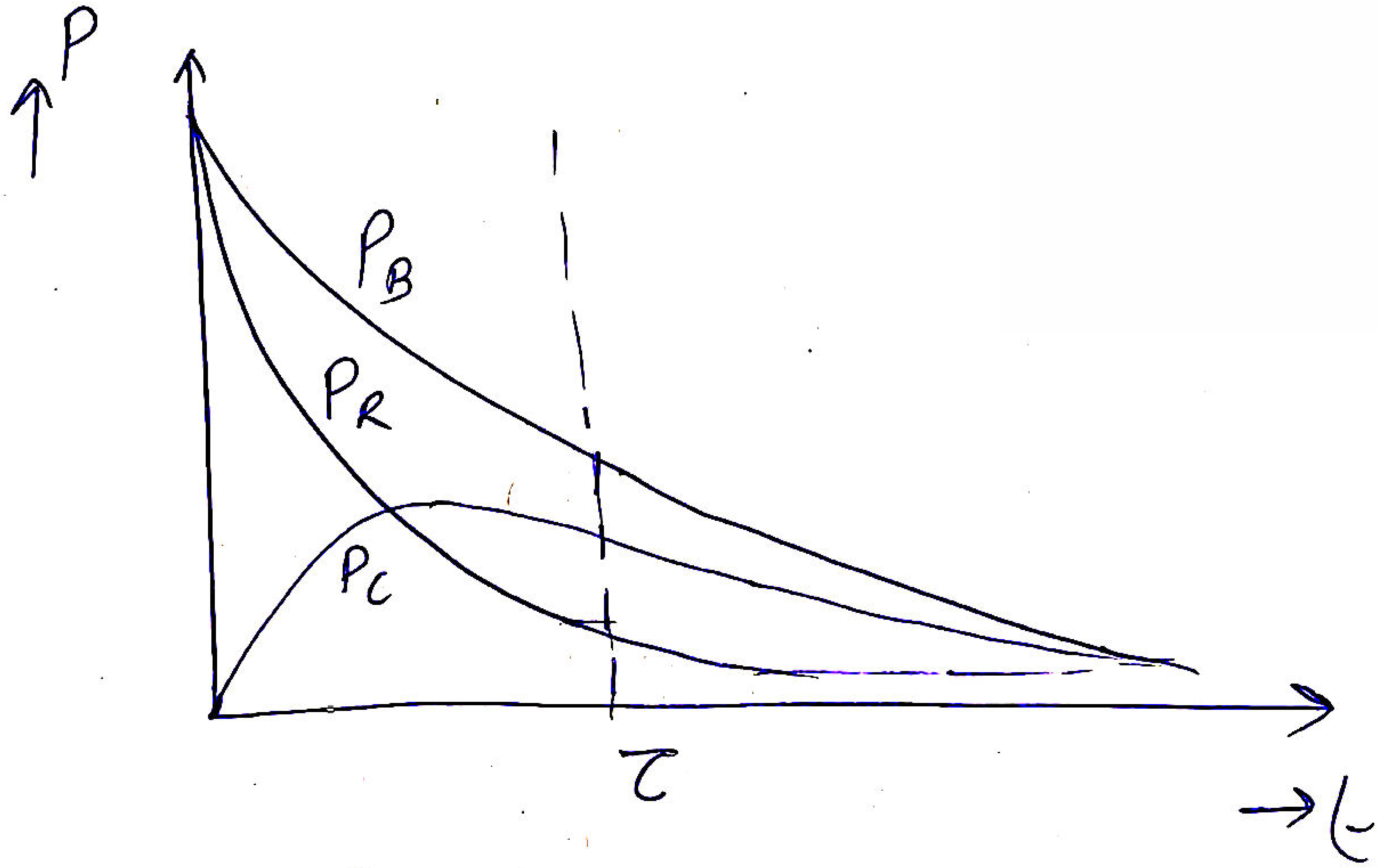
$$P_c = \frac{EC\left[1 - e^{-t/\tau}\right]}{C} \frac{E}{R} e^{-t/\tau}$$

$$\boxed{P_c = \frac{E^2}{R}\left[1 - e^{-t/\tau}\right] e^{-t/\tau}}$$

$P_B \rightarrow$ power delivered by battery

$= E i$

$$P_B = \frac{E^2}{R} e^{-t/\tau}$$



$P_R = i^2 R = \left[\frac{E}{R} e^{-t/\tau}\right]^2 R$

↳ power dissipated by resistor

$P_C \neq P_R$ at any time t

$\Rightarrow$ But $\boxed{P_R + P_C = P_B}$

$P_c \to max$

$$\frac{dP_c}{dt} = 0$$

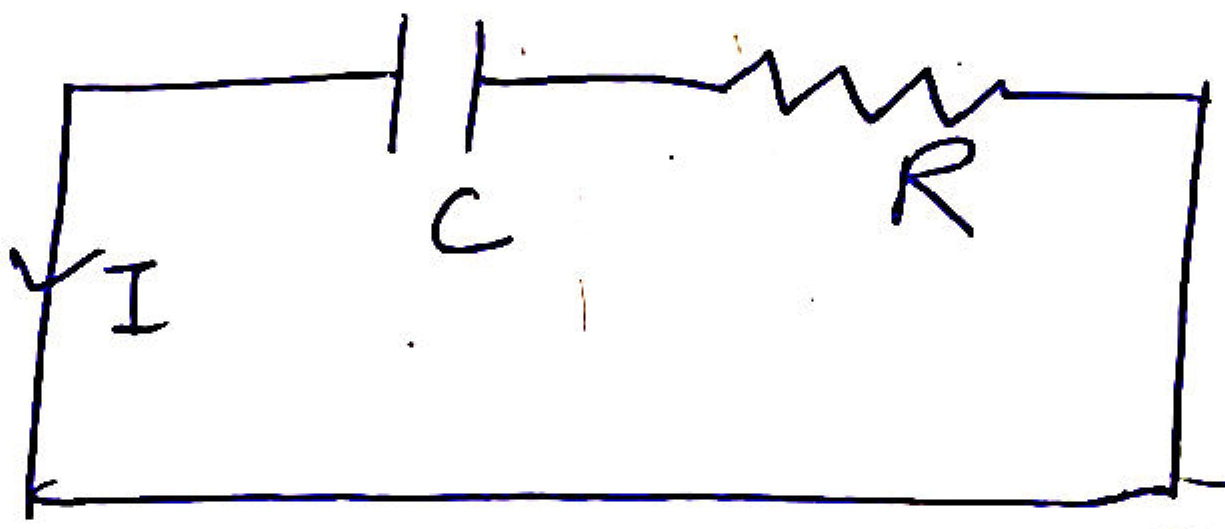
$$\frac{d}{dt}\left[\frac{E^2}{R}\left(e^{-t/\tau} - e^{-2t/\tau}\right)\right] = 0$$

$$e^{-t/\tau}\left[\frac{1}{\tau}\right] - e^{-2t/\tau}\left(\frac{2}{\tau}\right) = 0$$

$$\Rightarrow e^{t/\tau} = 2$$

or $\boxed{t = \tau \ln 2}$

For discharging RC circuit :



$q = Q e^{-t/\tau}$

$i = i_0 e^{-t/\tau}$

$\boxed{q = EC\, e^{-t/\tau}}$

$\boxed{Q = EC}$ $\boxed{i = \frac{E}{R} e^{-t/\tau}}$

$P_c = \frac{dU}{dt}$ rate at which energy is transffered

$$P_c = \frac{du}{dt} = \frac{d}{dt}\left[\frac{q^2}{2C}\right]$$

$$= \frac{d}{dt}\left[\frac{E^2C^2}{2C} e^{-2t/\tau}\right]$$

$$= \frac{E^2C}{2}\left[e^{-2t/\tau}\left[\frac{-2}{\tau}\right]\right]$$

$$\boxed{P_C = -\frac{E^2}{R} e^{-2t/\tau}}$$

P_R = energy dissipation by resistor

$$P_R = I^2R$$

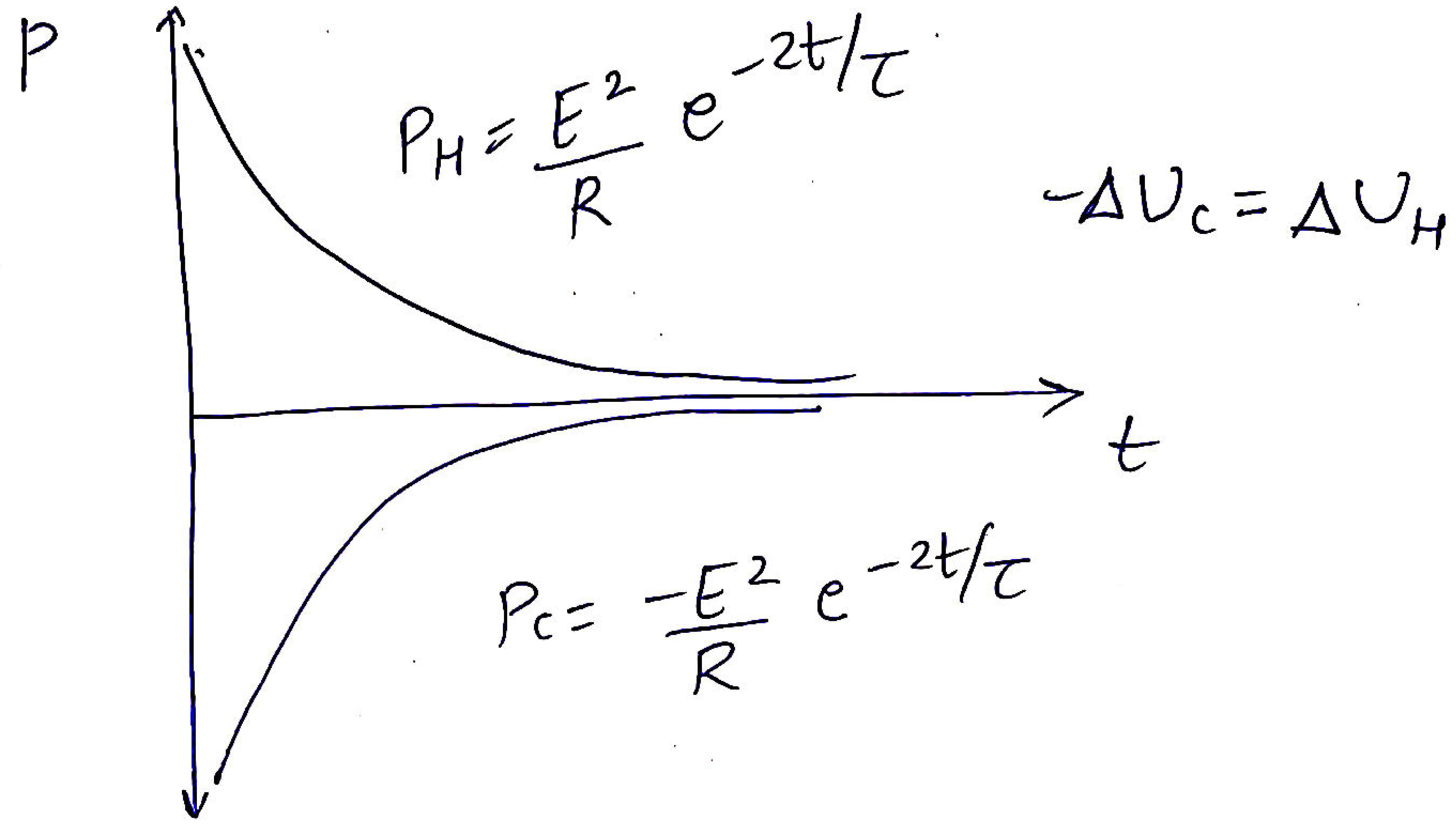
$$= \frac{E^2}{R^2} e^{-2t/\tau} R$$

$$\boxed{P_R = \frac{E^2}{R} e^{-2t/\tau}}$$

$\Rightarrow$ $P_C = -P_R$

or rate of energy decrease in capacitor

energy dissipated by capacitor will be lost by resistor.


P
$P_H = \frac{E^2}{R} e^{-2t/\tau}$
$-\Delta U_c = \Delta U_H$
t
$P_c = \frac{-E^2}{R} e^{-2t/\tau}$